The Hong Kong Institution of Engineers Transactions, Vol 18, No 2, pp26-33
Paper A024-201004; Received 15 April 2010; Accepted 6 August 2010

Dynamic Performance of Double-deck Bridges under Road Vehicles and Monorail Trains

SI Xuetong
Department of Civil Engineering, The University of Hong Kong

Francis T K AU
Department of Civil Engineering, The University of Hong Kong

GUO Wenhua
School of Civil Engineering and Architecture, Central South University

With the development of transportation network gaining momentum as prompted by modernisation, the design of bridges as crucial components of the network has become more complicated not only in respect of the more challenging structural forms but also the increasing functions to serve. While safety is acknowledged to be the most important, there are many other criteria to address, including aesthetics, efficiency, reliability, riding comfort, etc. Bridges that carry mixed modes of traffic pose a new problem as different vehicles perform differently.

This paper describes a framework for spatial dynamic analysis of double-deck bridges carrying both road vehicles and monorail trains taking into account road surface roughness and both vertical and lateral track irregularities. Both road vehicles and monorail trains are modelled as spatial mass-spring-damper systems while the bridge is modelled by finite elements. Then the governing equation of the whole interaction system can be assembled using a fully computerised approach and solved by direct integration. The dynamic performance of an authentic double-deck long-span bridge carrying both road vehicles and monorail trains is investigated with special emphasis on the riding comfort of vehicles and trains.

Keywords: *Coupled Systems, Double-deck Bridges, Monorail Trains, Riding Comfort, Road Vehicles, Vehicle-bridge Interaction*

Introduction

With the development of transportation network gaining momentum as prompted by modernisation of various countries, the design of bridges as crucial components of the network has become more complicated not only in respect of the more challenging structural forms but also the increasing functions to serve. While safety is acknowledged to be the most important, there are many other criteria to address, including aesthetics, efficiency, reliability, riding comfort, etc. To overcome various geographical barriers considered insurmountable until recently, numerous long-span bridges have been designed and built, including suspension bridges, cable-stayed bridges, steel truss bridges, arch bridges, etc. In order to optimise the use of materials and space, more and more bridges have been designed to carry both highway and railway traffic, such as Tsing Ma Bridge (1,377 m span, the HKSAR, China), Caiyuanba Bridge (800 m span, Chongqing, China) and the Messina Strait Bridge being planned (3,300 m span, Italy).

Relatively little research work has been done on the dynamic performance of bridges carrying monorail systems as compared to other modes of railway transportation. Actually the monorail system is an ideal short-distance railway system tailored for small to medium volume of passengers with the following advantages: (a) low cost of construction; (b) lower noise of monorail systems with rubber wheels compared with the traditional wheel-rail system; and (c) environmental friendliness and high reliability (Si, 2007).

The presence of heavy road vehicles and railway trains running on a long-span bridge may significantly change its dynamic behaviour and affect the fatigue life of the bridge. The oscillating bridge may in turn affect the safety and riding comfort of the road vehicles and trains moving on it. Therefore, the dynamic interaction among the road vehicles, the trains and the bridge becomes an important problem to address.

Various researchers have worked on the performance of vehicle-bridge interaction systems. Zaman *et al.* (1996) studied the dynamic response of cable-stayed bridges under moving vehicles modelled as single suspension units using the structural impedance method, with the bridge deck modelled as a plate without roughness and the cables modelled as springs. Green and Cebon (1997) discussed the importance of dynamic interaction between heavy vehicles and highway bridges, and presented a new method to investigate a simply-supported bridge traversed by a single degree-of-freedom (DOF) vehicle. A framework was developed by Cai and Chen (2004) to study the coupled 3-dimensional vehicle-bridge system under strong winds. It has been found that the driving speeds mainly affect the relative vertical responses of vehicles but not their rolling responses. The absolute response of vehicle is dominated by the bridge response when wind speed is high, while it is dominated by the road roughness when wind speed is low. Kim *et al.* (2005) later proposed a 3-dimensional modal analysis method to obtain the dynamic response of a steel girder bridge under moving vehicles, with the bridge modelled by finite elements with lumped masses and Rayleigh damping. Kwasniewski *et al.* (2006) studied the dynamic interaction between a heavy truck and a highway bridge using the commercial package LS-DYNA which use a detailed finite element truck model with 3-dimensional suspension systems, pneumatic and rotating wheels, with appropriate contact algorithms implemented. Guo and Xu (2000; 2001; 2002; 2003) conducted a series of research on the interaction between road vehicles and cable-stayed bridges. They proposed a fully computerised approach to study the dynamic responses of such vehicle-bridge systems and estimate the riding comfort of vehicles on bridges under crosswind.

As for the coupled train-bridge systems, Song *et al.* (2003) proposed a new method for 3-dimensional analysis of high-speed train-bridge interaction, with various improved elements to model the structural components of the bridge. Au *et al.* (2001; 2002) investigated the impact factors of cable-stayed bridges under moving highway and railway traffic to investigate the effects of different rail irregularities and damping. The study of Yau *et al.* (1999) on the impact response of high speed railway bridge and riding comfort of railway cars shows that the rail irregularity, ballast stiffness and suspension damping can drastically affect the riding comfort of railway cars travelling over simple beams while their effects are comparatively small for continuous beams.

For analysis of monorail bridges, Lee *et al.* (2005) introduced an analytical procedure for traffic-induced vibration of a 3-dimensional monorail train-bridge interaction system and investigated the riding comfort of monorail trains. Their group (Lee *et al.*, 2006) also studied

the dynamic responses of a monorail steel bridge under a moving train but they mainly focused on short-span simply supported bridges. Yu (2006) analysed the spatial coupled dynamic characteristics of a track girder of straddle monorail of simply supported span length ranging from 18 m to 26 m. Guo and Si (2007) later presented a method for dynamic analysis of coupled monorail train-bridge system in time domain with emphasis on simply supported bridges.

Most of the previous research efforts have been expended on the effects of either road vehicles or railway trains but not both. This paper describes a framework for 3-dimensional dynamic analysis of long-span bridges under moving road vehicles and monorail trains. The road vehicles and monorail trains are idealised as a series of masses with suspension system moving at different speeds. The bridge is modelled by finite elements with the consistent mass system and Rayleigh damping. The governing equation of motion of the coupled system is then assembled using the fully computerised approach considering the effect of vertical and lateral track irregularities as well as bridge deck roughness. An authentic double-deck long-span bridge carrying both road vehicles and monorail trains is analysed using the proposed framework.

Development of the Interaction System

Bridge Model

A bridge can be modelled by finite element method using spatial beam elements, cable elements, truss elements, etc as appropriate based on a global coordinate system comprising X-, Y- and Z-axes respectively along the longitudinal, lateral and vertical directions of the bridge deck following the right-hand rule. The mass distribution and structural damping should also be modelled in the interaction system for dynamic analysis. Usually consistent mass matrices are used for proper representation of the mass distribution.

To simulate the interaction system accurately, one of the most critical parameters is the damping coefficient. The most common approach is to use Rayleigh damping, where the damping matrix $[C_b]$ is assumed to be proportional to a linear combination of the stiffness matrix $[K_b]$ and mass matrix $[M_b]$ (Wilson, 2002), namely:

$$[C_b] = \alpha[M_b] + \beta[K_b] \tag{1}$$

Where α and β are the mass and stiffness multipliers respectively, which can be calculated by two arbitrary frequencies making use of the orthogonality properties of the mass and stiffness matrices. Here, they are obtained by the first two natural frequencies as:

$$\alpha = \frac{2(\xi_1\omega_2 - \xi_2\omega_1)}{\omega_2^2 - \omega_1^2}\,\omega_1\omega_2 \tag{2}$$

$$\beta = \frac{2(\xi_1\omega_2 - \xi_2\omega_1)}{\omega_2^2 - \omega_1^2} \tag{3}$$

Where ω_1 and ω_2 are the first two natural frequencies, while ξ_1 and ξ_2 are the corresponding damping ratios of the first two modes of the bridge. When both ξ_1 and ξ_2 are assumed to be equal to ξ, Eqn (2) and Eqn (3) can be simplified as:

$$\alpha = \frac{2\xi\omega_1\omega_2}{\omega_2 + \omega_1} \tag{4}$$

$$\beta = \frac{2\xi}{\omega_2 + \omega_1} \tag{5}$$

Finally, the transient response of the bridge is governed essentially by:

$$[M_b]\{\ddot{v}_b\} + [C_b]\{\dot{v}_b\} + [K_b]\{v_b\} = \{P_{be}\} \tag{6}$$

Where $\{\ddot{v}_b\}$, $\{\dot{v}_b\}$ and $\{v\}$ are the nodal dynamic acceleration, velocity and displacement vectors of the bridge respectively, and $\{P_{be}\}$ is the corresponding force vector.

Road-vehicle Model

To investigate the interaction between the road vehicles and the bridge, each vehicle is modelled as a combination of several rigid bodies connected by several spring and damping devices. Fig 1 shows a 3-dimensional model of a heavy road vehicle with 7 DOFs consisting of a car body with motion of bouncing Z_{vc}, pitching θ_{vc}, rolling ϕ_{vc}, and four tyres with only motion of bouncing Z_{vt}. The parameters K, M and C in general denote the spring constant, mass and damping coefficient respectively. The subscript vc denotes the car body of the vehicle; i indicates the lane number where the vehicle runs; and j indicates the serial number of the vehicle in the vehicular motorcade.

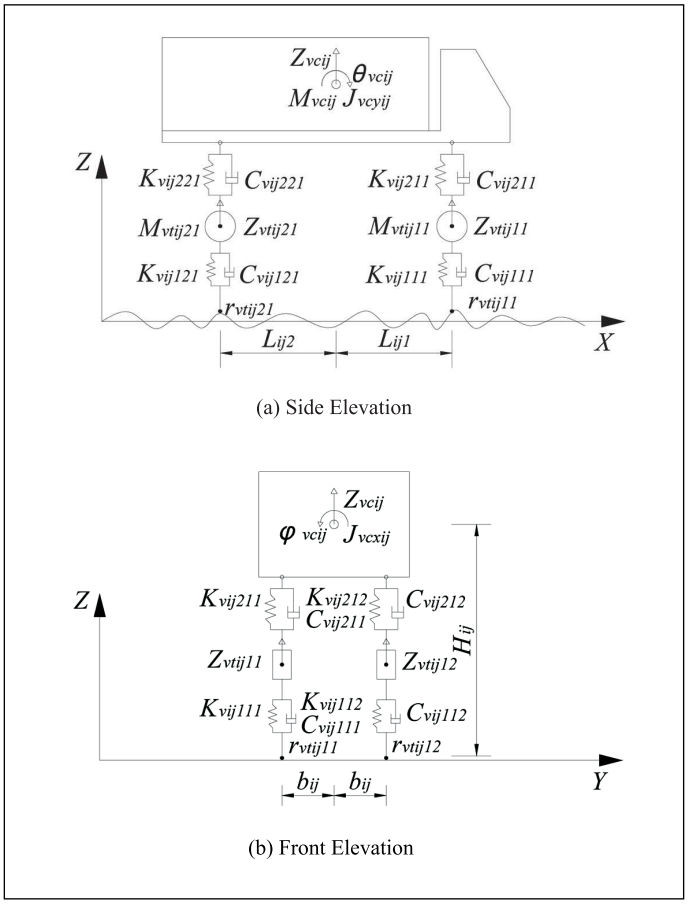


Figure 1 – An Idealised Spatial Model of a Heavy Road Vehicle with 7 DOFs

The assumptions made in this study are: (a) the elastic behaviour of car body is not considered; (b) the vehicle speed is constant while the path is straight; (c) the suspension springs are linearly elastic and the damping of the suspension system is viscous; and (d) the bridge deck and tyres of vehicles are in good contact without separation.

The relative spring deformation of air suspension can be expressed as:

$$R_{vij2kl} = Z_{vcij} + (-1)^k L_{ijk}\theta_{vcij} + (-1)^l b_{ij}\phi_{vcij} - Z_{vtijkl} \tag{7}$$

While the relative spring deformation at each tyre can be obtained by:

$$R_{vij1kl} = Z_{vtijkl} - V_{btijkl} - r_{btijkl} \tag{8}$$

Where r_{btijkl} is the bridge deck roughness at the contact point; subscripts vc and vt denote the car body and vehicular tyres respectively; k is the axle index (k = 1, 2 for the front and rear axle respectively); l indicates the tyre position in an axle (l = 1, 2 for the left and right positions of an axle respectively); V_{btijkl} denotes the vertical displacement of the bridge at the contact point. The vertical displacement V_{btijkl} of the bridge at the contact point is given by:

$$V_{btijkl} = [N_1 N_2 (e_{btijkl} N_5) N_3 N_4 (e_{btijkl} N_6)]_{x=x_c} [Z^{ei}\ \theta^{ei}\ \phi^{ei}\ Z^{ej}\ \theta^{ej}\ \phi^{ej}]^T \quad (9)$$

Where N_1, N_2, N_3, N_4, N_5 and N_6 are the usual shape functions of the spatial beam element; Z^{ei}, θ^{ei} and ϕ^{ei} are, respectively, the vertical displacement and rotations about Y- and X- axes of the corresponding beam element at Node i, while Z^{ej}, θ^{ej} and ϕ^{ej} are those at Node j; and e is the horizontal distance between the contact point and centre of the corresponding bridge element.

The deformation rates of the spring can be obtained by differentiation of the relative deformation as:

$$\dot{R}_{vij2kl} = \dot{Z}_{vcij} + (-1)^k L_{ijk} \dot{\theta}_{vcij} + (-1)^l b_{ij} \dot{\phi}_{vcij} - \dot{Z}_{vtijkl} \quad (10)$$

$$\dot{R}_{vij1kl} = \dot{Z}_{vtijkl} - N_1(x_c)\dot{Z}^{ei}_{btijkl} - N_2(x_c)\dot{\theta}^{ei}_{btijkl} - e_{btijkl} N_5(x_c)\dot{\phi}^{ei}_{btijkl} - N_3(x_c)\dot{Z}^{ej}_{btijkl} - N_4(x_c)\dot{\theta}^{ej}_{btijkl} - e_{btijkl} N_6(x_c)\dot{\phi}^{ej}_{btijkl} - \frac{\partial r_{btijkl}(x)}{\partial x} U_{vij} \quad (11)$$

Where U_v is the speed of vehicle and the over-dot denotes differentiation with respect to time t.

Monorail Train Model

A monorail train is supported on two bogies with each bogie having pneumatic travelling, steering and stabilising wheels that firmly grasp the track girder.

Fig 2 shows an idealised railcar model with 15 DOFs using the same coordinate system as before. The bouncing, swaying, pitching, rolling and yawing motion of the body and each bogie have been incorporated into the model. The track girder and wheels are assumed to be in good contact and there is no concentrated mass at each contact point.

The vertical and lateral relative spring deformations of air suspension can be written respectively as:

$$R_{mij2kl1} = Z_{mcij} + (-1)^k \theta_{mcij} L_{ijk} + (-1)^l \phi_{mcij} b_{ij1} - Z_{mbijk} - (-1)^l \phi_{mbijk} b_{ij2} \quad (12)$$

$$R_{mij2k12} = Y_{mcij} - (-1)^k \varphi_{mcij} L_{ijk} + \phi_{mcij} h_{ij1} - Y_{mbijk} + \phi_{mbijk} h_{ij2} \quad (13)$$

While the relative spring deformations at the driving, steering and stabilising wheels can be obtained respectively as:

$$R_{mdijklp} = Z_{mbijk} + (-1)^p \theta_{mbijk} L_{ij3} + (-1)^l \phi_{mbijk} b_{ij3} - V_{bdijklp} - r_{bdijklp} \quad (14)$$

$$R_{mtijklp} = Y_{mbijk} - (-1)^p \varphi_{mbijk} L_{ij4} + \phi_{mbijk} h_{ij3} - V_{btijklp} - r_{mtijklp} \quad (15)$$

$$R_{mbijklp} = Y_{mbijk} + \varphi_{mbijk} h_{ij4} - V_{bbijklp} - r_{mbijklp} \quad (16)$$

Where $V_{bdijklp}$, $V_{btijklp}$ and $V_{bbijklp}$ denote the displacements of the bridge at the contact point respectively; $r_{bdijklp}$, $r_{btijklp}$ and $r_{bbijklp}$ denote the surface roughness values at the contact points of driving, steering and stabilising wheels respectively; subscripts md, mt and mb denote the driving, steering and stabilising wheels respectively; i is the lane number where the monorail trains run; j is the serial number of the train in the motorcade; k denotes the position of the bogie; p denotes the wheel position in a bogie (p = 1, 2 for the front and rear sides respectively); and index l denotes the wheel position in an axle (l = 1, 2 for the left and right sides respectively).

The displacements of the bridge at the contact points can be calculated as:

$$V_{bdijklp} = [N_1 N_2 (e_{bdijklp} N_5) N_3 N_4 (e_{bdijklp} N_6)]_{x=x_c} [Z^{ei}\ \theta^{ei}\ \phi^{ei}\ Z^{ej}\ \theta^{ej}\ \phi^{ej}]^T_{bdijklp} \quad (17)$$

$$V_{btijklp} = [N_1 N_2 (h_{btijklp} N_5) N_3 N_4 (e_{btijklp} N_6)]_{x=x_c} [Y^{ei}\ \varphi^{ei}\ \phi^{ei}\ Y^{ej}\ \varphi^{ej}\ \phi^{ej}]^T_{btijklp} \quad (18)$$

$$V_{bbijklp} = [N_1 N_2 (h_{bbijklp} N_5) N_3 N_4 (e_{bbijklp} N_6)]_{x=x_c} [Y^{ei}\ \varphi^{ei}\ \phi^{ei}\ Y^{ej}\ \varphi^{ej}\ \phi^{ej}]^T_{bbijklp} \quad (19)$$

Where e and h are the horizontal and vertical distances between the contact point and centre of the corresponding bridge element respectively.

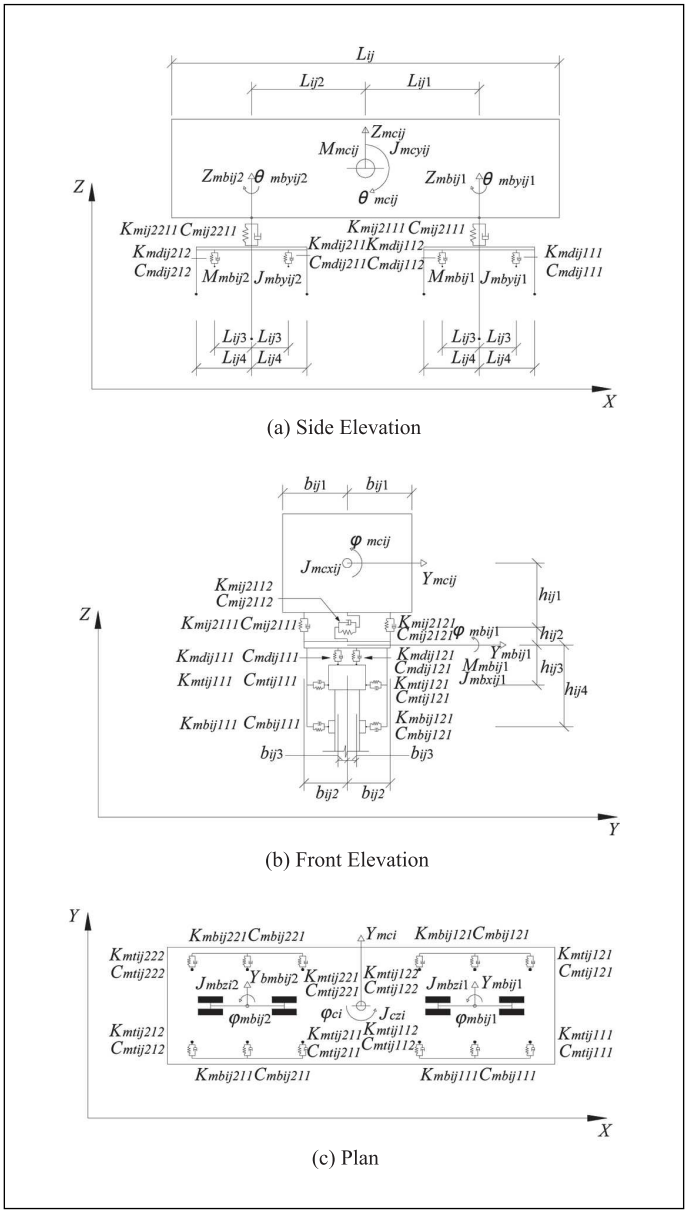


Figure 2 – Idealised Model of a Monorail Train with 15 DOFs

The deformation rate of each spring can be obtained by differentiation of the relative deformation with respect to time t as:

$$\dot{R}_{mij2kl1} = \dot{Z}_{mcij} + (-1)^k \dot{\theta}_{mcij} L_{ijk} + (-1)^l \dot{\phi}_{mcij} b_{ij1} - \dot{Z}_{mbijk} - (-1)^l \dot{\phi}_{mbijk} b_{ij2} \quad (20)$$

$$\dot{R}_{mij2k12} = \dot{Y}_{mcij} - (-1)^k \dot{\varphi}_{mcij} L_{ijk} + \dot{\phi}_{mcij} h_{ij1} - \dot{Y}_{mbijk} + \dot{\phi}_{mbijk} h_{ij2} \quad (21)$$

$$\dot{R}_{mdijklp} = \dot{Z}_{mbijk} + (-1)^p \dot{\theta}_{mbijk} L_{ij3} + (-1)^l \dot{\phi}_{mbijk} b_{ij3} - N_1(x_c)\dot{Z}^{ei}_{bdijklp} - N_4(x_c)\dot{\theta}^{ej}_{bdijklp} - e_{bdijklp} N_5(x_c)\dot{\phi}^{ei}_{bdijklp} - e_{bdijklp} N_6(x_c)\dot{\phi}^{ej}_{bdijklp} - N_2(x_c)\dot{\theta}^{ei}_{bdijklp} - N_3(x_c)\dot{Z}^{ej}_{bdijklp} - \frac{\partial r_{bdijklp}(x)}{\partial x} U_{mij} \quad (22)$$

$$\dot{R}_{mtijklp} = \dot{Y}_{mbijk} + (-1)^p \dot{\varphi}_{mbijk} L_{ij4} + \dot{\phi}_{mbijk} h_{ij3} - N_1(x_c)\dot{Y}^{ei}_{btijklp} - N_2(x_c)\dot{\varphi}^{ei}_{btijklp} - h_{btijklp} N_5(x_c)\dot{\phi}^{ei}_{btijklp} - h_{btijklp} N_6(x_c)\dot{\phi}^{ej}_{btijklp} - N_4(x_c)\dot{\varphi}^{ej}_{btijklp} - N_3(x_c)\dot{Y}^{ej}_{btijklp} - \frac{\partial r_{btijklp}(x)}{\partial x} U_{mij} \quad (23)$$

$$\dot{R}_{mbijklp} = \dot{Y}_{mbijk} + \dot{\phi}_{mbijk} h_{ij4} - N_1(x_c)\dot{Y}^{ei}_{bbijklp} - N^*_2(x_c)\dot{\varphi}^{ei}_{bbijklp} - h_{bbijklp} N_5(x_c)\dot{\phi}^{ei}_{bbijklp} - h_{bbijklp} N_6(x_c)\dot{\phi}^{ej}_{bbijklp} - N^*_4(x_c)\dot{\varphi}^{ej}_{bbijklp} - N_3(x_c)\dot{Y}^{ej}_{bbijklp} - \frac{\partial r_{bbijklp}(x)}{\partial x} U_{mij} \quad (24)$$

Where U_m is the speed of monorail trains.

Road Surface and Track Irregularities

The surface roughness of a bridge is one of the important factors that excite a vehicle-bridge system (Yau *et al.*, 1999; Au *et al.*, 2002; Cai and Chen, 2004). It can be taken as a zero-mean stationary Gaussian random process and simulated by spectral representation based on the specified power spectral density (PSD) (Guo and Xu, 2001), namely:

$$r(x) = \sum_{k=1}^{N} \sqrt{2S(\bar{\Omega}_k)\Delta\bar{\Omega}}\cos(2\pi\Delta\bar{\Omega}_k x + \theta_k) \tag{25}$$

Where θ_k is a random phase angle complying with a uniform distribution from 0 to 2π, and $S(\bar{\Omega}_k)$ is the PSD of road surface roughness.

The PSD function used for the road surface roughness is (Guo and Xu, 2001):

$$S_v(\bar{\phi}) = A_r(\bar{\phi}/\bar{\phi}_0)^{-2} \tag{26}$$

Where $\bar{\phi}$ is the spatial frequency; $\bar{\phi}_0 = 1/2\pi$ is a non-continuous frequency coefficient; and A_r is the surface roughness coefficient of bridge deck. The PSD function adopted for the vertical and lateral track irregularities is (Lee *et al.*, 2005; Lee *et al.*, 2006):

$$S_{z0}(\Omega) = \frac{\alpha_m}{\Omega^n + \beta_m^n} \tag{27}$$

Where $\Omega = \omega/2\pi$ is the spatial frequency; α, β and n are, respectively, the roughness coefficient, shape parameter and a parameter to express the power distribution of the PSD.

Interaction System

Assuming that all the relevant displacements remain small, the principle of virtual work is used to derive the governing equation of the coupled system. The equilibrium condition of the bridge under its self-weight without any vehicle is taken as the initial state. Then the virtual work δW done by inertial forces, damping forces, elastic forces and external loading can be obtained, thereby giving:

$$\delta W_I^v + \delta W_I^m + \delta W_I^b + \delta W_D^v + \delta W_D^m + \delta W_D^b + \delta W_S^v + \delta W_S^m + \delta W_S^b + \delta W_{Ex}^b = 0 \tag{28}$$

In which

$$\delta W_I^b = \sum_{i=1}^{n_b}\sum_{j=1}^{n_b} m_{ij}\ddot{v}_j\delta v_i \tag{29}$$

$$\delta W_D^b = \sum_{i=1}^{n_b}\sum_{j=1}^{n_b} c_{ij}\dot{v}_j\delta v_i \tag{30}$$

$$\delta W_S^b = \sum_{i=1}^{n_b}\sum_{j=1}^{n_b} k_{ij}v_j\delta v_i \tag{31}$$

$$\delta W_I^v = \sum_{i=1}^{L_v}\sum_{j=1}^{n_{vi}}(M_{vcij}\ddot{Z}_{vcij}\delta Z_{vcij} + J_{vcxij}\ddot{\phi}_{vcij}\delta\phi_{vcij} + J_{vcyij}\ddot{\theta}_{vcij}\delta\theta_{vcij} + \sum_{k=1}^{2}\sum_{l=1}^{2} M_{vtijkl}\ddot{Z}\delta Z_{vtijkl}) \tag{32}$$

$$\delta W_D^v = \sum_{i=1}^{L_v}\sum_{j=1}^{n_{vi}}\sum_{k=1}^{2}\sum_{l=1}^{2}(C_{vij2kl}\dot{R}_{vij2kl}\delta R_{vij2kl} + C_{vij1kl}\dot{R}_{vij1kl}\delta R_{vij1kl}) \tag{33}$$

$$\delta W_S^v = \sum_{i=1}^{L_v}\sum_{j=1}^{n_{vi}}\sum_{k=1}^{2}\sum_{l=1}^{2}(K_{vij2kl}R_{vij2kl}\delta R_{vij2kl} + K_{vij1kl}R_{vij1kl}\delta R_{vij1kl}) \tag{34}$$

$$\delta W_I^m = \sum_{i=1}^{L_m}\sum_{j=1}^{n_{mi}}\{M_{mcij}(Y_{mcij}\delta Y_{vcij} + \ddot{Z}_{mcij}\delta Z_{mcij}) + J_{mcxij}\ddot{\phi}_{mcij}\delta\phi_{mcij} + J_{mcyij}\ddot{\theta}_{mcij}\delta\theta_{mcij} + J_{mczij}\ddot{\varphi}_{mcij}\delta\varphi_{mcij} + \sum_{k=1}^{2}[M_{mbijk}(\ddot{Y}_{mbijk}\delta Y_{mbijk} + Z_{mbijk}\delta Z_{mbijk}) + J_{mbxijk}\ddot{\phi}_{mbijk}\delta\phi_{mbijk} + J_{mbyijk}\ddot{\theta}_{mbijk}\delta\theta_{mbijk} + J_{mbzijk}\ddot{\varphi}_{mbijk}\delta\varphi_{mbijk}]\} \tag{35}$$

$$\delta W_D^m = \sum_{i=1}^{L_m}\sum_{j=1}^{n_{mi}}\sum_{k=1}^{2}\sum_{l=1}^{2}[C_{mij2kl1}\dot{R}_{mij2kl1}\delta R_{mij2kl1} + C_{mij2kl2}\dot{R}_{mij2kl2}\delta R_{mij2kl2}D_{1l} + \sum_{p=1}^{2}(C_{mdijklp}\dot{R}_{mdijklp}\delta R_{mdijklp} + C_{mtijklp}\dot{R}_{mtijklp}\delta R_{mtijklp} + C_{mbijklp}\dot{R}_{mbijklp}\delta R_{mbijklp}D_{1p})] \tag{36}$$

$$\delta W_S^m = \sum_{i=1}^{L_m}\sum_{j=1}^{n_{mi}}\sum_{k=1}^{2}\sum_{l=1}^{2}[K_{mij2kl1}R_{mij2kl1}\delta R_{mij2kl1} + K_{mij2kl2}R_{mij2kl2}\delta R_{mij2kl2}D_{1l} + \sum_{p=1}^{2}(K_{mdijklp}R_{mdijklp}\delta R_{mdijklp} + K_{mtijklp}R_{mtijklp}\delta R_{mtijklp} + K_{mbijklp}R_{mbijklp}\delta R_{mbijklp}D_{1p})] \tag{37}$$

$$\delta W_{Ex}^b = \sum_{i=1}^{L_v}\sum_{j=1}^{n_{vi}}\sum_{k=1}^{2}\sum_{l=1}^{2}F_{vtijkl}\delta Z_{btijkl} + \sum_{i=1}^{L_m}\sum_{j=1}^{n_{mi}}\sum_{k=1}^{2}\sum_{l=1}^{2}\sum_{p=1}^{2}F_{mdijklp}\delta Z_{bdijklp} \tag{38}$$

$$F_{vtijkl} = \frac{g}{4}\left(M_{vcij} + \sum_{k=1}^{2}\sum_{l=1}^{2}M_{vtijkl}\right) \tag{39}$$

$$F_{mdijklp} = \frac{g}{8}\left(M_{mcij} + \sum_{k=1}^{2}M_{mbijk}\right) \tag{40}$$

Where the subscripts I, D, S and Ex stand for the inertial, damping, elastic and external effects respectively; the subscripts v, m and b denote the vehicle, monorail train and bridge respectively; n_b is the total number of bridge elements; L_v and L_m are, respectively, the total number of lanes of road vehicles and monorail trains; n_m and n_v are the total number of monorail trains and road vehicles respectively; F_{vtijkl} and F_{mdijkl} are the gravitational forces on the bridge at the location of each tyre of road vehicle and of each driving wheel of monorail train, respectively, which are taken as the average weight per wheel as appropriate; and D_{ij} is the Kronecker Delta function.

Finally the governing equation of the coupled system can be obtained by means of the fully computerised approach (Guo and Xu, 2001) as:

$$[M_{bmv}]\{\ddot{v}_{bmv}\} + [C_{bmv}]\{\dot{v}_{bmv}\} + [K_{bmv}]\{v_{bmv}\} = \{P_{bmv}\} \tag{41}$$

In which

$$\{v_{bmv}\} = \begin{Bmatrix} v_b \\ v_m \\ v_v \end{Bmatrix}$$

$$[M_{bmv}] = \begin{bmatrix} M_b + M_{bbm} + M_{bbv} & 0 & 0 \\ 0 & M_m & 0 \\ 0 & 0 & M_v \end{bmatrix}$$

$$[C_{bmv}] = \begin{bmatrix} C_b + C_{bbm1} + C_{bbv1} & C_{bm1} & C_{bv1} \\ C_{mb1} & C_m + C_{m1} & 0 \\ C_{vb1} & 0 & C_v + C_{v1} \end{bmatrix}$$

$$[K_{bmv}] = \begin{bmatrix} K_b + K_{bbm1} + K_{bbv1} & K_{bm1} & K_{bv1} \\ K_{mb1} & K_m + K_{m1} & 0 \\ K_{vb1} & 0 & K_v + K_{v1} \end{bmatrix}$$

$$\{P_{bmv}\} = \begin{Bmatrix} \begin{bmatrix} P_{bmg} + P_{bmrl} + P_{bmr2} + P_{bmr3} \\ + P_{vbg} + P_{bvrl} + P_{bvr2} + P_{bvr3} \end{bmatrix} \\ P_{mr2} + P_{mr3} \\ P_{vr2} + P_{vr3} \end{Bmatrix}$$

Where $[M_{bmv}]$, $[C_{bmv}]$ and $[K_{bmv}]$ are the mass, damping and stiffness matrices of the coupled system respectively; $\{v_b\}$, $\{v_m\}$ and $\{v_v\}$ are the displacement vectors of the bridge, monorail trains and road vehicles respectively; $[M_b]$, $[C_b]$ and $[K_b]$ are the mass, damping and stiffness matrices of the bridge

respectively; $[K_m]$ and $[C_m]$ are the stiffness and damping matrices of the monorail trains respectively; $[K_v]$ and $[C_v]$ are the stiffness and damping matrices of road vehicles respectively; $[M_{bbm}]$ and $[M_{bbv}]$ account for the inertia forces of masses, respectively, of monorail trains and vehicles at contact points due to the bridge accelerations; $[C_{bm1}]$ and $[C_{mb1}]$ account for the damping of suspension systems of monorail trains while $[C_{bv1}]$ and $[C_{vb1}]$ account for those of vehicles; $[K_{bm1}]$, $[K_{mb1}]$, $[K_{bv1}]$ and $[K_{vb1}]$ account for the corresponding stiffness of the suspension systems; $[C_{bbm1}]$ and $[C_{bbv1}]$ account for the additional effects on the bridge damping $[C_b]$ due to suspension systems of monorail trains and vehicles respectively; $[K_{bbm1}]$ and $[K_{bbv1}]$ account for the corresponding additional effects on the bridge stiffness $[K_b]$; $[C_{m1}]$ and $[C_{v1}]$ account for the additional effects of suspension on damping of monorail trains and road vehicles respectively; $[K_{m1}]$ and $[K_{v1}]$ account for the corresponding additional stiffness effects; $[P_{bmg}]$ and $[P_{bvg}]$ denote the loading from monorail trains and vehicles on the bridge; $[P_{bmr1}]$ and $[P_{bvr1}]$ account for the inertia forces of masses, respectively, of monorail trains and vehicles due to road roughness at contact points; $[P_{bmr2}]$, $[P_{bvr2}]$, $[P_{bmr3}]$ and $[P_{bvr3}]$ are additional force vectors acting on the bridge caused by the interaction between suspension and surface roughness; $[P_{mr2}]$ and $[P_{vr2}]$ account for the loading on monorail trains and vehicles respectively due to their damping at contact points and surface roughness; and $[P_{mr3}]$ and $[P_{vr3}]$ are force vectors acting on monorail trains and vehicles respectively due to their springs at contact points and surface roughness.

Numerical Studies

Prototype Bridge

Caiyuanba Yangtze River Bridge in Chongqing is the largest long-span steel tied arch bridge carrying both road vehicles and monorail trains. It has an overall length of 800 m with a main span of 420 m and two side spans of 102 m and 88 m on each side (Fig 3). Road vehicles run on the upper deck while monorail trains run on the track girders at the bottom chord of the truss. The locations A, B, C and D in Fig 3 are those where further results will be worked out.

A 3-dimensional finite element model is established as shown in Fig 4. The arch ribs and transverse connections between arch ribs are modelled by spatial beam elements, while the suspenders are modelled by spatial truss element. Spatial beam elements are also adopted for the steel truss, main piers and Y-shape rigid frames, etc. The connections between the bridge components and supports are also properly modelled.

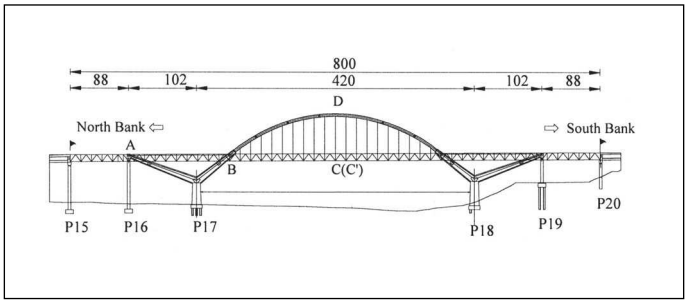


Figure 3 – Caiyuanba Yangtze River Bridge in Chongqing (Dimensions in m)

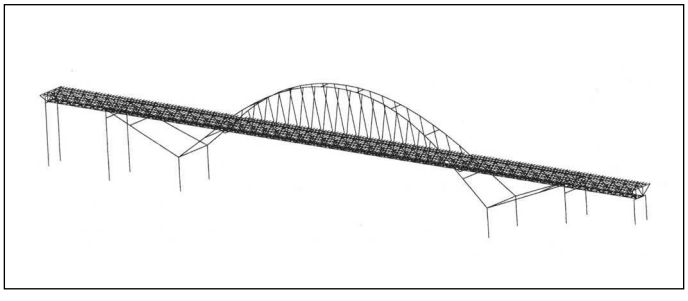


Figure 4 – Finite Element Model of Caiyuanba Bridge

Mode	Bridge	Vehicle	Monorail Train
1	0.254	1.806	0.973
2	0.268	1.863	2.280
3	0.288	3.291	2.293
4	0.304	4.989	2.819
5	0.310	5.451	3.401

Table 1 – The First Five Natural Frequencies of the Components of the Coupled System (Hz)

The first five natural frequencies of each component sub-system of the coupled system can be obtained using subspace iteration method as shown in Table 1. It shows that the fundamental frequency of monorail train is closer to that of the bridge as compared to the vehicle. So it is more likely for the monorail train to excite the bridge.

Simulation of Surface Roughness of Bridge Deck and Irregularities of Monorail Track

The surface roughness of the bridge deck and irregularities of the monorail track girder are simulated by spectral representation. The roughness coefficient A_r in Eqn (26) is taken as 20×10^{-6} m^3/cycle according to International Organisation for Standardisation (ISO) specification for good roads (Guo, 2003). The simulation frequency range spans from 0.05 Hz to 2 Hz. The surface profiles of tracks for driving, steering and stabilising wheels are simulated using Eqn (27) with the following parameters (Lee *et al.*, 2005; Lee *et al.*, 2006): (a) driving: $\alpha = 0.0005$, $\beta = 0.35$, $n = 3.00$; (b) steering: $\alpha = 0.0006$, $\beta = 0.5$, $n = 2.80$; and (c) stabilising: $\alpha = 0.0006$, $\beta = 0.5$, $n = 2.60$ and the simulation frequency range goes from 0.01 Hz to 2 Hz. The simulated surface roughness profile for the bridge deck and irregularities of monorail track over a length of 3,000 m are shown in Fig 5.

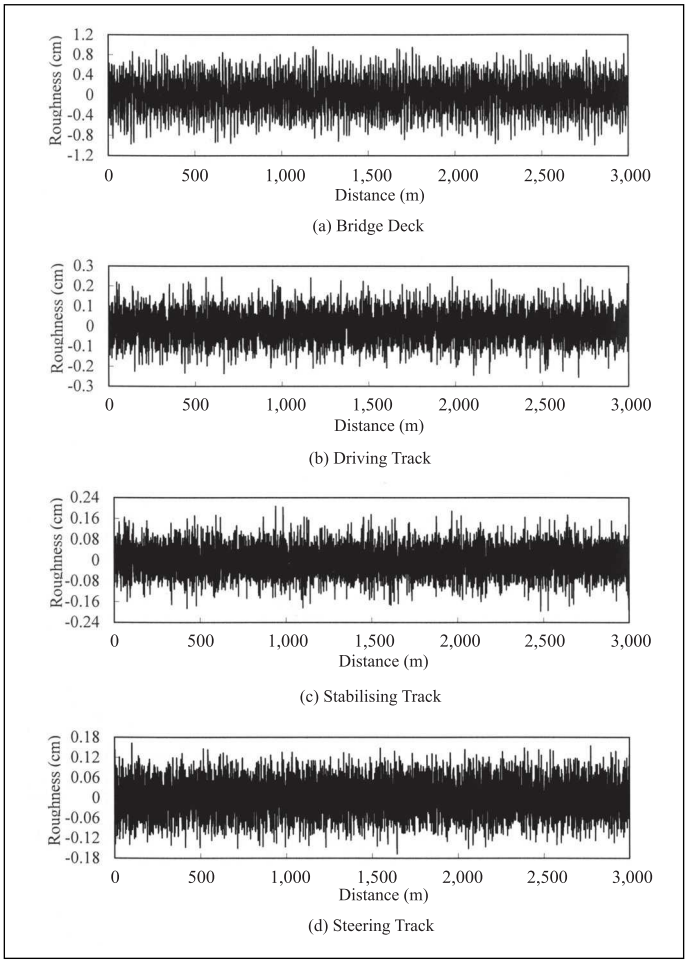


Figure 5 – Surface Roughness of Bridge Deck and Irregularities of Monorail Track

Bridge Responses

The dynamic responses of the coupled system are investigated under three cases, namely: (a) Case SC: a motorcade of 20 heavy road vehicles at centre to centre spacing of 21 m runs in Lane 2 (Fig 6(a)); (b) Case SM: a monorail train with eight railcars at centre to centre spacing of 15.6 m runs on the left track (Fig 6(b)); and (c) Case SMSC: both the motorcade in Case SC and the monorail train in Case SM run simultaneously. In the analysis, the damping ratios of the bridge are taken as 1%. Wilson-θ method is adopted to solve the transient differential equations of this interaction system with a time interval of 0.01 s. The speed is taken as 75 km/h. All the parameters of road vehicles and monorail trains adopted here are the same as those in Si (2007).

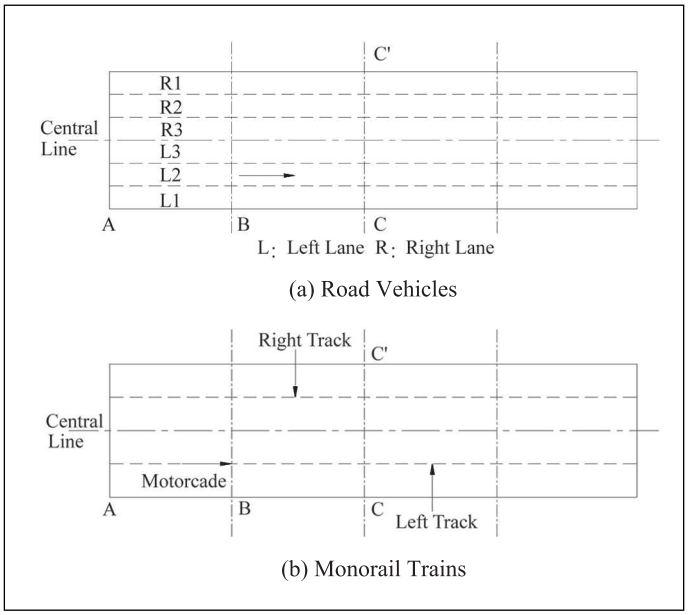


Figure 6 – Live Load on the Bridge: (a) Road Vehicles; and (b) Monorail Trains

Because of page limitation, only the results at the middle of main span (ie Point C) are presented in this paper. The variations of vertical and lateral displacements at Point C at the middle of main span as shown in Fig 7 demonstrate that the dynamic responses of the bridge under both moving monorail trains and road vehicles are bigger than those under either monorail trains or road vehicles alone. The dynamic responses of the bridge are also significantly influenced by the monorail trains when both types of traffic run on the bridge simultaneously. The relatively small deflections at point C in various cases as compared with the long span of the bridge show that the bridge has very good stiffnesses both vertically and laterally.

The time history of acceleration at point C as shown in Fig 8 indicates that the overall time history of acceleration of Case SM almost overlaps with that of Case SMSC. The bridge is seen to be more sensitive to excitations by monorail trains than those by road vehicles. This can be further confirmed by noting that the natural frequency of monorail trains is closer to that of the bridge. Fig 8(b) demonstrates that the road vehicles do not excite the bridge much mainly because the vehicle model has no lateral DOFs.

Evaluation of Riding Comfort of Road Vehicles and Monorail Trains

After obtaining the dynamic responses of the bridge, the assessments of riding comfort in terms of acceleration of the locomotive in the monorail train and the first road vehicle in the motorcade of all three cases are studied according to ISO 2631-1 (ISO, 1997). One of the crucial factors governing riding quality is the duration of vibration to which the passengers are subject (Dukkipati *et al.*, 2008). Normally the longer the duration of vibration exposure, the lower the tolerable vibration. Figs 9 and 10 show that the root mean square accelerations in the one-

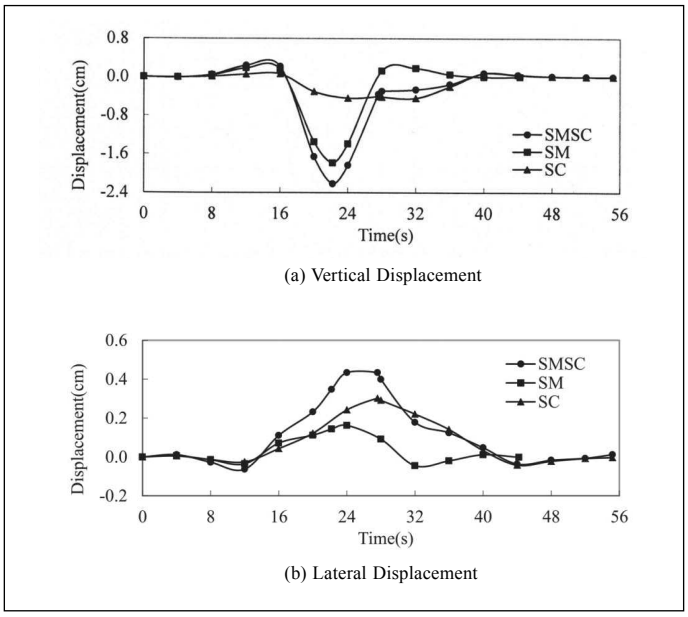


Figure 7 – Displacements of Point C at the Middle of Main Span

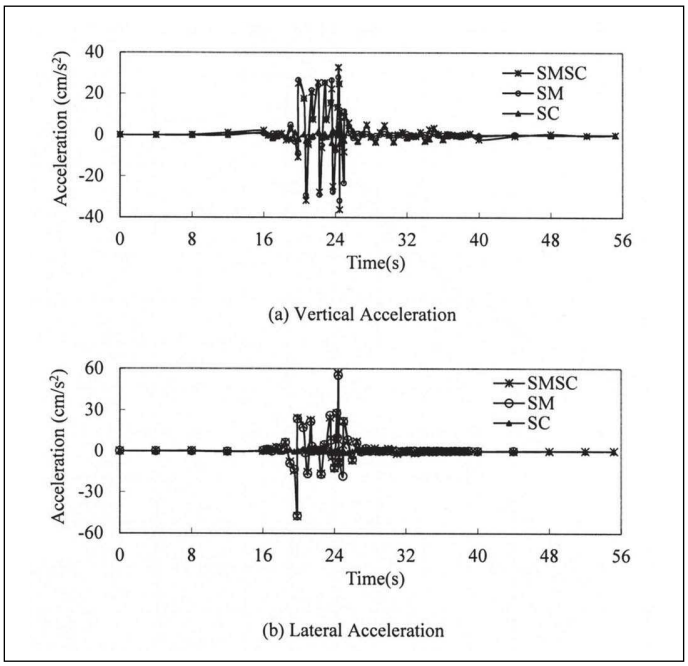


Figure 8 – Time History of Acceleration of Point C at the Middle of Main Span

third octave band of both the locomotive and the first road vehicle of all cases are lower than the most sensitive allowable frequency values (ie 4 Hz to 8 Hz for vertical vibrations and below 2 Hz for lateral vibrations), which indicates that their riding comfort can be classified as good. The accelerations are all considered acceptable with reference to the tolerable limits up to one hour duration.

Fig 9(a) shows that, when monorail trains move on the bridge, the riding comfort of the first road vehicle is reduced in the range of lower frequencies without affecting it much in the range of higher frequencies. However, Fig 9 also shows that, compared with Case SM, the vehicle motorcade has no obvious effect on the riding comfort of the locomotive when they run on the bridge simultaneously. Fig 9(b) indicates that passengers feel more comfortable in the monorail trains than in the heavy road vehicles in Case SMSC when both run simultaneously. The vertical and lateral accelerations of locomotive shown in Fig 10 further confirm that the riding comfort of monorail trains is little affected by the road vehicles.

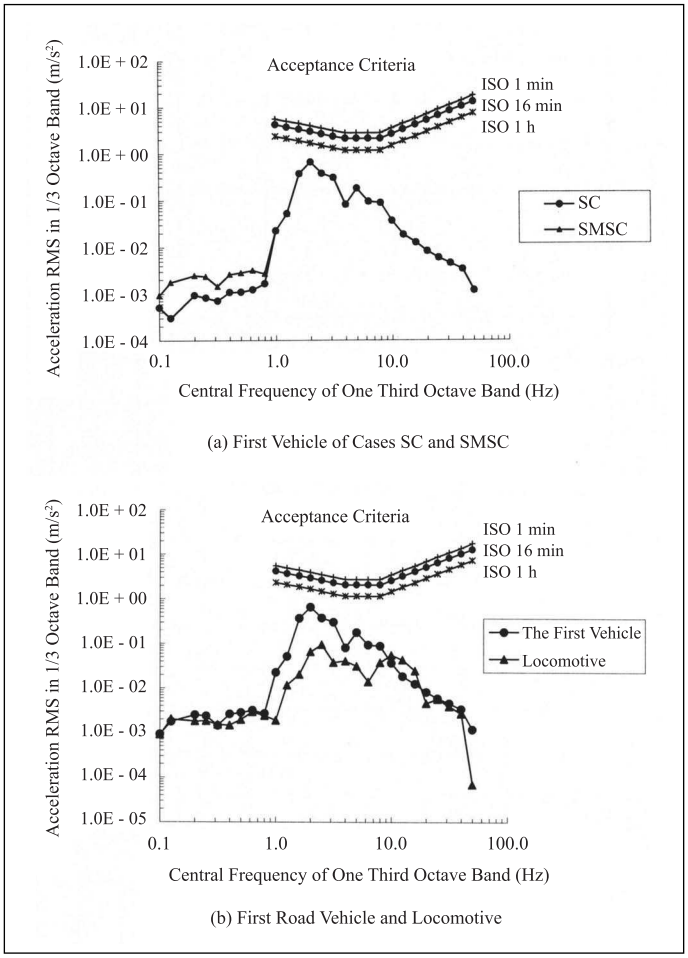


Figure 9 – Riding Comfort in the Vertical Direction of: (a) First Vehicle of Cases SC and SMSC; and (b) First Road Vehicle and Locomotive of Case SMSC

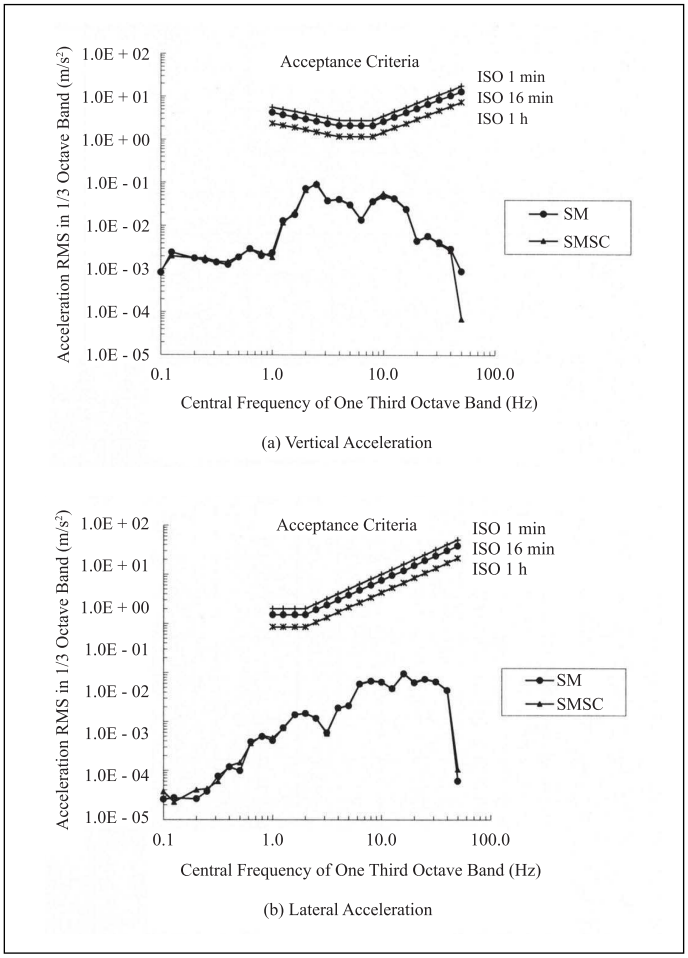


Figure 10 – Riding Comfort of the Locomotive of Cases SM and SMSC

Conclusion

An analytical framework for dynamic analysis of the coupled system consisting of double-deck long-span bridges, road vehicles and monorail trains is presented considering different types and arrangement of vehicles and trains running on different lanes of the bridge. The vertical and lateral irregularities of monorail track girders and random roughness of bridge deck are taken into account in the formulation. The coupled system can be simplified into either a vehicle-bridge or train-bridge interaction system. The proposed framework together with the ISO 2631-1 specification provides a systematic method to investigate the dynamic performance of each component sub-system in an interaction system and evaluate the riding comfort of the road vehicles or monorail trains.

The Caiyuanba Yangtze River Bridge in Chongqing that carries both road vehicles and monorail trains has been chosen as a case study to demonstrate the use of the proposed framework in investigation of the dynamic behaviour. The results show that: (a) the dynamic responses of the bridge increase with the number of road vehicles and/or monorail trains on the bridge; (b) the monorail trains have more effects on the dynamic performance of the long-span bridge than road vehicles, and the responses of the bridge are more sensitive to the movement of monorail trains when both road vehicles and monorail trains run on the bridge simultaneously; (c) the riding comfort of road vehicles and monorail trains running on the chosen bridge can be estimated as good in various cases; and (d) the monorail is a comfortable transportation system due to the adoption of rubber tyres. Besides, in the case studied, the monorail trains can reduce the riding comfort of road vehicles in the range of lower frequencies. Therefore a comprehensive study of riding comfort is crucial to the design of long-span bridges carrying mixed modes of traffic.

Acknowledgements

The authors gratefully acknowledge the financial support provided by the National Natural Science Foundation of China (Project No 51078356) and the Research Grants Council (RGC) of the HKSAR, China (RGC Project No HKU 710208E).

References

1. Au, F.T.K., Cheng, Y.S., Cheung, Y.K., Effect of random road roughness and long-term deflection of prestressed concrete girder and cable-stayed bridges on impact due to moving vehicles. *Computers and Structures*. Volume 79, pp853-872. (2001).
2. Au, F.T.K., Wang, J.J., Cheung, Y.K., Impact study of cable-stayed railway bridges with random rail irregularities. *Engineering Structures*. Volume 24, pp529-541. (2002).
3. Cai, C.S., Chen, S.R., Framework of vehicle-bridge-wind dynamic analysis. *Journal of Wind Engineering and Industrial Aerodynamics*. Volume 92, pp579-607. (2004).
4. Dukkipati, R., Pang, J., Qatu, M., Sheng, G., Zuo, S., *Road vehicle dynamics*. SAE International. Warrendale, Pa (2008).
5. Green, M.F., Cebon, D., Dynamic interaction between heavy vehicles and heavy bridges. *Computers and Structures*. Volume 62, No 2, pp253-264. (1997).
6. Guo, W.H., *Dynamic analysis of coupled road vehicle and long span cable-stayed bridge systems under cross winds*. PhD Thesis. The Hong Kong Polytechnic University. Hong Kong (2003).
7. Guo, W.H., Si, X.T., *Dynamic performance of simply-supported bridges under moving monorail trains.* Second International Conference on Structural Condition Assessment, Monitoring and Improvement. Changsha, China (2007).
8. Guo, W.H., Xu, Y.L., Direct assembling matrix method for dynamic analysis of coupled vehicle-bridge system. *Proceedings of International Conference on Advances in Structural Dynamics.* Hong Kong (2000).

9. Guo, W.H., Xu, Y.L., Fully computerised approach to study cable-stayed bridge-vehicle interaction. *Journal of Sound and Vibration*. Volume 248, No 4, pp745-761. (2001).
10. Guo, W.H., Xu, Y.L., Evaluation of ride comfort of road vehicles running on a cable-stayed bridge under crosswind. *Proceedings of the Third International Conference on Advances in Steel Structures*. Hong Kong (2002).
11. Guo, W.H., Xu, Y.L., Dynamic analysis of moving road vehicle under a sudden crosswind. *Key Engineering Materials*. Volume 243, pp123-128. (2003).
12. ISO 2631-1, Mechanical vibration and shock – evaluation of human exposure to whole-body vibration, Part I: general requirement. (1997).
13. Kim, C.W., Kawatani, M., Kim, K.B., 3-dimensional dynamic analysis for bridge-vehicle interaction with roadway roughness. *Computers and Structures*. Volume 83, pp1627-1645. (2005).
14. Kwasniewski, L., Li, H.Y., Wekezer, J., Malachowski, J., Finite element analysis of vehicle-bridge interaction. *Finite Elements in Analysis and Design*. Volume 42, pp950-959. (2006).
15. Lee, C.H., Kawatani, M., Kim, C.W., Nishimura, N., Kobayashi, Y., Dynamic response of a monorail steel bridge under a moving train. *Journal of Sound and Vibration*. Volume 294, pp562-579. (2006).
16. Lee, C.H., Kim, C.W., Kawatani, M., Nishimura, N., Kamizono, T., Dynamic response analysis of monorail bridges under moving trains and riding comfort of trains. *Engineering Structures*. Volume 27, pp1999-2013. (2005).
17. Si, X.T., *Dynamic performance and riding-comfort evaluation of a coupled system of double-deck long-span bridges, road vehicles and monorail trains*. MPhil Thesis. Central South University. Changsha, China (2007).
18. Song, M.K., Noh, H.C., Choi, C.K., A new 3-dimensional finite element analysis model of high-speed train-bridge interaction. *Engineering Structures*. Volume 25, pp1611-1626. (2003).
19. Wilson, E.L., 3-dimensional Static and Dynamic Analysis of Structures (3rd). Berkeley, Computers and Structures Inc. (2002).
20. Yau, J.D., Yang, Y.B., Kuo, S.R., Impact response of high speed rail bridges and riding comfort of rail cars. *Engineering Structures*. Volume 21, pp836-844. (1999).
21. Yu, Z.H., *Study on spatial coupling dynamics characteristics of vehicle-guideway in track traffic system of straddle monorail*. MPhil Thesis. Beijing Jiaotong University. Beijing, China (2006).
22. Zaman, M., Taheri, M.R, Khanna, A., Dynamic response of cable-stayed bridges to moving vehicles using the structural impedance method. *Appl Math Modelling*. Volume 20, pp877-889. (1996).

About the Authors



SI Xuetong **BEng MPhil**
Email: tonysi@hku.hk

Mr Si obtained his BEng degree with first class honours and MPhil degree at Central South University. Currently, he is a PhD candidate of the University of Hong Kong. His research interests include time-dependent analysis of concrete structures considering concrete creep, shrinkage and cable relaxation, dynamic analysis of concrete structures taking into account time-dependent behaviour, vehicle-bridge interaction and wind effects on structures.



Francis T K AU **BSc(Eng) MSc(Eng) PhD FHKIE FIStructE MICE R.P.E. CEng 1RSE (PRC)**
Email: francis.au@hku.hk

Ir Prof Au received his BSc(Eng) degree with first class honours, MSc(Eng) degree with distinction and PhD degree from the University of Hong Kong. He joined Maunsell Consultants Asia as an engineer upon graduation with his first degree, where he was involved in new town projects, highway projects as well as marine engineering projects. He returned to the University of Hong Kong in 1988 after having seven years of practical experience in the industry. He has been involved in various research and consultancy projects in bridge engineering and concrete structures.



GUO Wenhua **BEng MPhil PhD**
Email: whguo@mail.csu.edu.cn

Prof Guo obtained his BEng, MPhil and PhD degrees in Changsha Railway College, now part of Central South University. He also obtained a PhD degree in the Hong Kong Polytechnic University. His main research interests are wind effects on buildings and bridges, wind-vehicle-bridge interaction systems, seismic effects on bridges and bridge vibration and stabilisation.