

A Uniform Nonlinear Control Method for DC-DC Converters with Fast Transient Response

Much effort has been made to the switching surface control of buck converter. However, it remains a fundamental challenge to apply the same concepts and associated control techniques to converters with non-minimum-phase characteristics. A nonlinear control method for all basic DC-DC converters (ie buck converter, boost converter, buck-boost converter, Ćuk converter and SEPIC) is proposed by deriving a uniform switching surface. It retains the advantages of boundary control for buck converter with second-order switching surface. For example, the converters can reach steady state in two switching actions after being subjected to large-signal disturbances. The derivations are general-oriented and can determine the circuit variables needed in the switching function. The same controller is universally applicable to different converters operating in both continuous conduction and discontinuous conduction modes. The potential applications lie in power conversions which require fast response to line/load changes, such as the point of load (POL) converters in the distributed power conversion system, power conditioning stage of the conversion system supplied by renewable energy sources, and so on. Design procedures and performance of the proposed control method applied on a 190 W 48/48 V buck-boost converter prototype will presented and compared with theoretical predictions.

Keywords: Boundary Control, Switching Surface Control, Second-order Switching Surface, State Trajectory Control, Sliding Mode Control

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Introduction

DC-DC converters have been widely used in our daily consumer electronic products, information and telecom communication industry, electrical vehicles, renewable energy conversion system, medical instruments, aerospace and defense applications. Beside the requirements of designing DC-DC converters with high efficiency, high power density, low cost and high reliability over the last three decades, fast transient response corresponding to line/load changes is becoming more and more crucial in our current and future power conversion systems. One example is for the standalone applications in distributed power systems, which contains a lot of point of load (POL) converters connected in parallel, such as the power supply system in mass transit railway (MTR) of Hong Kong, the load change is frequently and generates high load current variations. Another example is conversion systems supplied by renewable energy sources, such as fuel cell and solar power, the input voltages are varied with operation time.

Much effort has been made in developing various control schemes for switched-mode DC-DC converters to achieve good output regulation and dynamic response. The controller is dominantly designed with small-signal linearisation techniques in frequency domain (Erickson and Maksimovic, 2001). However, switching converters are highly nonlinear dynamical systems and their large-signal characteristics will behave differently from that predicted by the small-signal design approaches. Furthermore, some converter circuits like boost- and buck-boost-derived converters are non-minimum phase systems, having a right-half-plane zero in the linearised control-to-output transfer function. They are slow in responding because of larger phase lag between input and output signals, resulting in faulty behaviour at the start of a response. This property makes the controller impossible to be designed with classical control theories to achieve fast dynamic response over wide bandwidth of supply and load disturbances.

To overcome the limitations of conventional small-signal-based controllers, an alternative concept is to design controllers directly in time domain to achieve fast dynamic responses. One major class of them is switching surface control. The concept is to determine the time sequences to turn

on/off switches according to certain constrains, namely, switching surfaces (Krein, 1998; Munzert and Krein, 1996; Greuel *et al.*, 1997; Krein, 2001). Typical switching surface control methods are hysteretic control with zero-order switching surface or sliding-mode control (SMC) with first-order switching surface (Utkin, 1974; Huang and Liu, 1989; Mattavelli *et al.*, 1993; Malesani *et al.*, 1995; Utkin and Sabanovic, 1999; Spiazzi and Mattavelli, 2002; Tan *et al.*, 2005; Tan *et al.*, 2006). The hysteretic controller tightly regulates the inductor current at the current reference. With further extension on regulating the output voltage, the SMC is the popular choice in boundary control. However, the optimal sliding surface and the stability for fast dynamic response depend on the supply and load characteristics. It is thus difficult to design a set of well-defined control parameters for the sliding surface at all operating points. The control parameters such as the slope of the switching function are sometimes designed by trial-and-error, start-up profile, switching frequency, etc. In general, the converter requires taking several switching actions before settling to the steady state. The SMC is sometimes applied to the fast control loop and the output is regulated by a proportional-integral (PI) controller. The PI controller is designed by classical control theory or sophisticated design method, like the fuzzy controller.

Instead of a linear switching surface, second-order and high-order switching surfaces are proposed for buck converter (Leung and Chung, 2004; Leung and Chung, 2005; Leung and Chung, 2007; Chiu *et al.*, 2007). These nonlinear switching surfaces are approximately or ideally follow the on/off state trajectories of buck converter. Therefore, the parameters used in the control law are well-defined and the converter can revert to steady-state after two switching actions. A single control law was applicable for buck converter operating in both continuous conduction mode (CCM) and discontinuous conduction mode (DCM). The control methods are named as boundary control with second-order/high-order switching surface. Apart from the aforementioned merits, there are several practical issues to apply boundary control with second/high-order switching surfaces to DC-DC converters which are also common issues for other switching surface control methods. For example, variable frequency under different operating line/load conditions, sensitivity of the parameters of output filter, high inductor current peak. To handle

these issues, several techniques have been proposed for buck-converter (Yan *et al.*, 2008; Babazadeh and Maksimović, 2009; Corradini *et al.*, 2009) to achieve fixed frequency, parameter independent solution or programmed inductor current. However, one remaining fundamental issue which poses great challenges is that the same concept cannot be easily applied to converters with non-minimum-phase characteristics (Redl and Sun, 2009). It is difficult to formulate a simple switching surface on the state-plane for a converter with state trajectories in spiral shape, such as the boost converter discussed by Song and Chung (2008).

The objective of this paper is to tackle the above mentioned fundamental issue in switching surface control, therefore, to apply the concept of boundary control with second-order switching surface to all of basic DC-DC converters and make it possible to apply the associated developed control techniques for buck converter to other kind of converters. Accordingly, a uniform second-order switching surface σ^2 on a Cartesian x-y plane rather than the state plane is proposed. On the Cartesian x-y plane, switching surfaces of basic DC-DC converters have the same form like that of buck converter and the derivations are based on a generalised procedure. Partial of the preliminary results have been given in a previous paper (Wang *et al.*, 2009). The comprehensive analysis and verifications of the proposed control method will be presented here. The uniform controller exhibits the following properties: 1) it retains the advantages of boundary control with second-order switching surface for buck converter, such as fast dynamic response and well-defined control parameters; 2) the same controller is applicable for different type of converters with the input variables to the controller being modified accordingly, implying the feasibility to manufacture universal analogue controller ICs or provide universal digital solutions for different converters; 3) the switching surface for each converter is well-defined and does not need to modify the structure of power stage, like the ones (Viswanathan *et al.*, 2005; Calvente *et al.*, 2004; Krein, 2006).

The following section describes the concept of the proposed control scheme and its stability analysis. Simulation results to verify its universal applicability to basic DC-DC converters are also given. And then the derivation procedures of implementation of the proposed uniform control method will be illustrated by a buck-boost converter. Experimental results of a 48/48 V 190 W prototype will also be presented to verify the feasibility to apply the proposed controller to basic DC-DC converters.

Uniform Second-order Switching Surface σ^2

Derivation of Uniform Second-order Switching Surface

Fig 1 shows the basic DC-DC converters. The first three ones, as presented in Figs 1(a) to 1(c), are composed of one capacitor and one inductor. The last two, Ćuk converter and SEPIC, include two inductors and two capacitors. Fig 2 plots the theoretical waveforms of their inductor currents and capacitor voltages. Generally, there are two types of distinctive curves

to present the relationship between inductor currents and capacitor voltages as shown in Figs 2(a) and 2(b), respectively. It can be noticed from Fig 2(b) that the capacitor voltages are monotonically decrease with the corresponding inductor currents for converters with non-minimum phase characteristics, which is the implication of the difficulty in applying switching surface control to those converters.

To handle the above fundamental issue in switching surface control and incorporate the advantages of the boundary control with second-order switching surface which has been applied on buck converter (Leung and Chung, 2004), a uniform second-order switching surface is proposed. It extends the control state variables (ie inductor currents and capacitor voltages) in previously proposed switching surface controllers to arbitrary direct or indirect control variables, namely x and y. Consequently, the relationships between x and y in all basic DC-DC converters are exactly the same as that of i_L - v_C in buck converter shown in Fig 2(a). Therefore, the switching surfaces of different kind of converters can be easily defined on a Cartesian x-y plane. x and y have the same properties as those of inductor current and capacitor voltage respectively in buck converter, implying that:

$$\frac{dy}{dx} = 2k_s (x - x_{ref}) \quad (1)$$

Where k_s is state trajectory parameter related with the slope of x and x_{ref} is the reference value.

With the aid of Fig 3, according to Eqn (1), the approximated on-state trajectory $Traj|_{on}$ and off-state CCM trajectory $Traj|_{off}$ can be expressed respectively as follows:

$$Traj|_{on} = y(t) - y(t_0) - k_{s1} \{ [x(t) - x_{ref}]^2 - [x(t_0) - x_{ref}]^2 \} = 0 \quad (2)$$

$$Traj|_{off} = y(t) - y(t_2) - k_{s2} \{ [x(t) - x_{ref}]^2 - [x(t_2) - x_{ref}]^2 \} = 0 \quad (3)$$

Where t_0 and t_2 are the turn on/off instants, respectively, k_{s1} is the trajectory parameter during on-state period and k_{s2} is that during off-state CCM period. $k_{s1} > 0$ and $k_{s2} < 0$. In DCM, after the x reaches zero, the trajectory moves along the y-axis, $x = 0$ and the corresponding trajectory parameter $k_{s3} = 0$ during this interval.

Fig 4 presents typical switching surface, load line and idea on/off state-trajectories of basic DC-DC converters on x-y plane. The trajectories are plotted by firstly solving on- and off-state space equations of specific converter with different initial conditions and then mapping the state variables i_L and v_C to x and y, respectively. It is the ideal state trajectories which can be approximated by Eqn (2) and Eqn (3).

As discussed by Leung and Chung (2004), the idea second-order switching surface σ^2 should pass through the target operating point (y_{ref}, x_{ref}), and exactly along the approximated on-state trajectory when x is below the load line, and the off-state trajectory when x is above the load line.

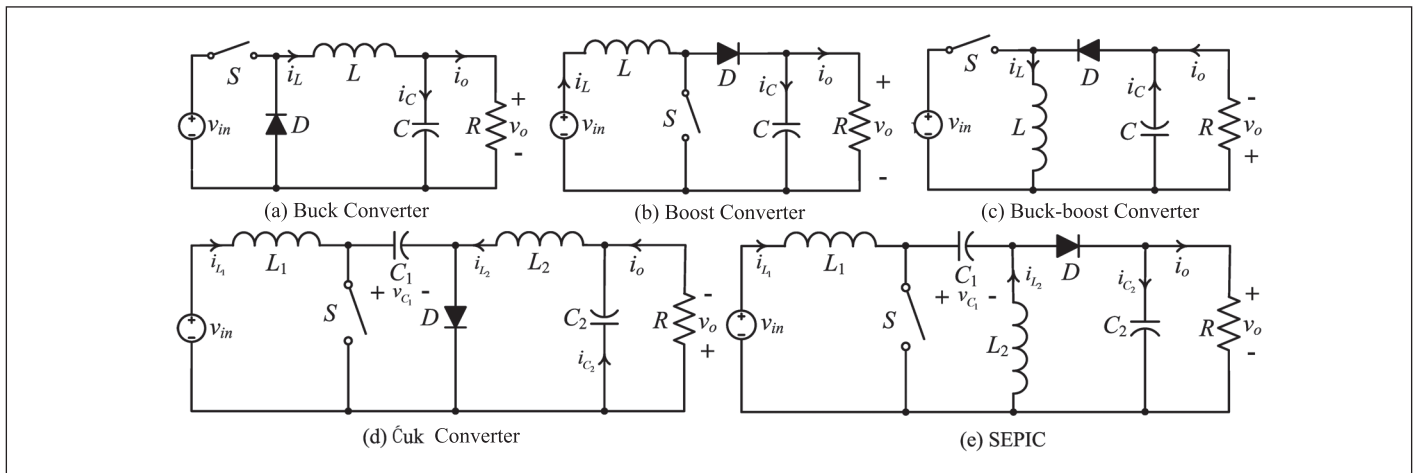


Figure 1 – Basic DC-DC Converters

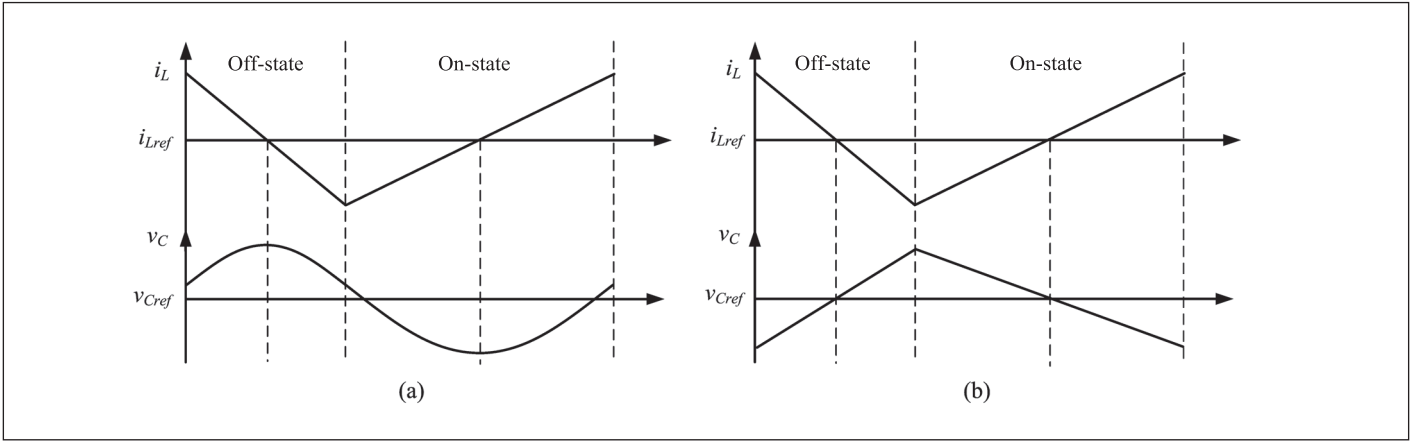


Figure 2 – Theoretical Relationships between Inductor Current and Capacitor Voltage: (a) i_L - v_C in Buck Converter and i_{L2} - v_{C2} in Ćuk Converter; (b) i_L - v_C in Boost and Buck-boost Converter, i_{L1} - v_{C1} in Ćuk Converter and i_{L1} - v_{C1} , i_{L2} - v_{C2} in SEPIC

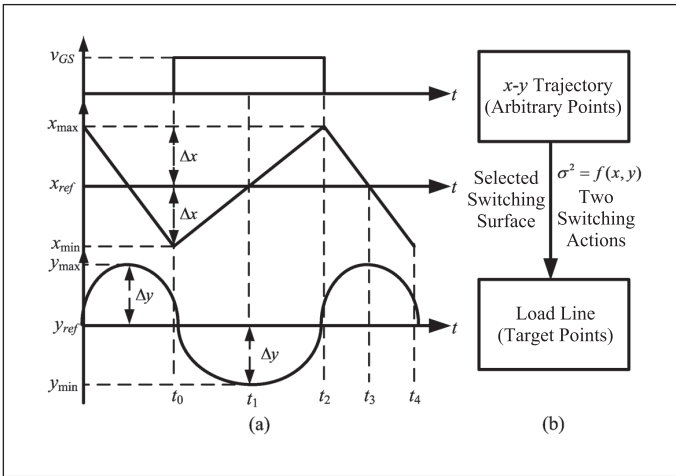


Figure 3 – Principle of Boundary Control with Uniform Second Order Switching Surface: (a) Control Variables, (b) Control Law

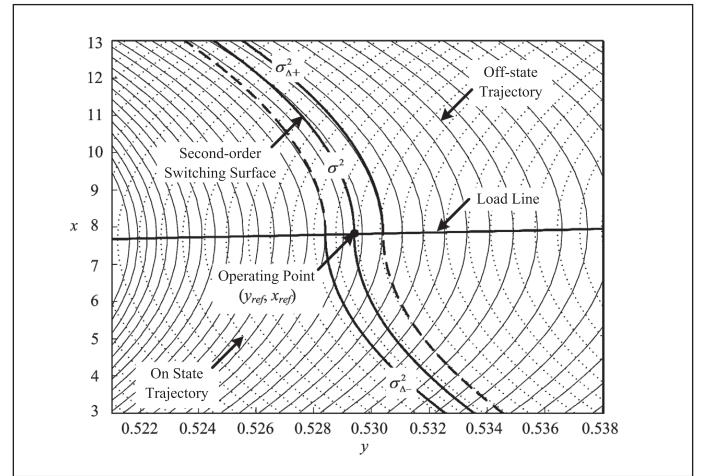


Figure 4 – Typical Trajectories, Load Line and Switching Surface of Basic DC-DC Converters on the x-y Plane

By transforming the i_L - v_C state plane to the Cartesian x-y plane, a well-defined uniform second-order switching surface σ^2 can be obtained as shown in Fig 4. It approximately follows the ideal on-state trajectory and off-state trajectory when the state is below and above the load line, respectively, implying a high velocity to revert to the target operating point from arbitrary operating points.

The switching instants are determined by predicting the operating point at t_1 and t_3 as shown in Fig 3. By putting $y(t) = y_{ref}$, $x(t) = x_{ref}$, $y(t_0) = y$, and $y(t_2) = y$ into Eqn (2) and Eqn (3), respectively, a general form of the switching surface is given by:

$$\sigma^2 = (y - y_{ref}) - k(x - x_{ref})^2 \quad (4)$$

Where σ^2 is the uniform second-order switching surface, k is the control parameter and can be expressed as follows for an ideal second-order switching surface:

$$k = k_{s1} \left[\frac{1 - \text{sgn}(x - x_{ref})}{2} \right] + k_{s2} \left[\frac{1 + \text{sgn}(x - x_{ref})}{2} \right] \quad (5)$$

k can be of other values, which affect the stability and trajectory velocity along the switching surface as discussed later. In the following analysis, k_1 and k_2 are defined as general control parameters for turn-on and turn-off switching actions, respectively. Practically, in order to avoid chattering phenomenon, a modified switching surface σ_{Δ}^2 is derived by adding a hysteresis band $2\Delta y$ as shown in Fig 4 into Eqn (4), resulting in:

$$\sigma_{\Delta}^2 = \begin{cases} \sigma_{\Delta-}^2 = y - y_{ref} - k_1(x - x_{ref})^2 + \Delta y & (x < x_{ref}) \\ \sigma_{\Delta+}^2 = y - y_{ref} - k_2(x - x_{ref})^2 - \Delta y & (x > x_{ref}) \end{cases} \quad (6)$$

Where $\sigma_{\Delta-}^2$ and $\sigma_{\Delta+}^2$ are the uniform second-order switching surface below and above load line.

Therefore, the uniform control law for the basic DC-DC converters is formulated as follows. The switch S is turned on if:

$$y(t) - k_1(x - x_{ref})^2 - (y_{ref} - \Delta y) \leq 0 \text{ and } x(t) < x_{ref} \quad (7a)$$

S is turned off if:

$$y(t) - k_2(x - x_{ref})^2 - (y_{ref} + \Delta y) \geq 0 \text{ and } x(t) > x_{ref} \quad (7b)$$

Table 1 and 2 tabulates the transformed variables x , y and their associated load line and control parameters k_{s1} and k_{s2} for ideal second-order switching surfaces. An example demonstrating the derivation procedures of x , y and k for a buck-boost converter will be given later in this paper. It should be noted that the derivations for Ćuk converter and SEPIC are based on reduced-order models discussed by Huang and Liu (1989), Mattavelli *et al.* (1993) and Malesani *et al.* (1995). In Ćuk converter, the output inductor current i_{L2} and output capacitor voltage v_{C2} are selected as the state variables. In SEPIC, the inductor currents i_{L1} , i_{L2} and output capacitor voltage v_{C2} are chosen as the state variables. The results for SEPIC presented in Table 1 and 2 are based on assumptions that the

voltage across the dc-blocking capacitor C_1 is equal to v_{in} , and the converter is designed with $L_1 = L_2 = L$.

Simulations have been done on boost converter, buck-boost converter, Ćuk converter and SEPIC to verify the validation of the results shown in Table 1 and 2. Load changes from CCM to CCM, from CCM to DCM and from DCM to CCM are simulated. The results presented in Fig 5 reveal that the single controller (with different control parameters) can control the four converters operating in both CCM and DCM and make them revert to steady-state by two switching actions after being exposed to large disturbances. It can also be noted from Figs 5(e) and 5(f) that x and y exhibit similar properties during steady-state and transient periods as those of i_L and v_C in buck converter discussed by Leung and Chung (2004).

Converter Type	Load Line	x	y
Buck	$i_L = \frac{v_o}{R}$	i_L	v_o
Boost	$i_L = \frac{v_o i_o}{v_{in}}$	i_L	$\frac{1}{2} Li_L^2 + \frac{1}{2} Cv_o^2$
Buck-boost	$i_L = \frac{v_o i_o}{v_{in}} + i_o$	i_L	$\frac{1}{2} Li_L^2 + \frac{1}{2} Cv_o^2 + Cv_{in} v_o$
Ćuk	$i_{L2} = \frac{v_o}{R}$	i_{L2}	v_o
SEPIC	$i_{L1} + i_{L2} = \frac{v_o i_o}{v_{in}} + i_o$	$i_{L1} + i_{L2}$	$\frac{1}{2} L(i_{L1} + i_{L2})^2 + C_2 v_o^2 + 2C_2 v_{in} v_o$

Table 1 – Load Line and Control Variables of the Proposed Uniform Control Law

Converter Type	k_{s1} (On-state)	k_{s2} (Off-state CCM)	k_{s2} (Off-state DCM)
Buck	$\frac{L}{2C(v_{in} - v_o)}$	$-\frac{L}{2Cv_o}$	0
Boost	$\frac{L}{2}$	$\frac{Lv_{in}}{2(v_{in} - v_o)}$	0
Buck-boost	$\frac{L}{2}$	$-\frac{Lv_{in}}{2v_o}$	0
Ćuk	$\frac{L_2}{2C_2(v_{C1} - v_o)}$	$-\frac{L_2}{2C_2v_o}$	0
SEPIC	$\frac{L}{2}$	$-\frac{Lv_{in}}{2v_o}$	0

Table 2 – Control Parameters of the Proposed Uniform Control Law

Stability Analysis

By varying the control parameters of k_1 and k_2 , the points along $\sigma_\Delta^2 = 0$ can be divided based on the directions of on/off trajectories at the switching surface (Krein, 2001). There are three possible modes of the points: rejective mode, reflective mode and refractive mode. In rejective mode, the trajectories on both sides of the switching surface depart away from the switching surface, which will lead to unstable operation of the converter. In reflective mode, the state trajectories direct toward the switching surface on both side and will move along the surface to the target operating point. In refractive mode, trajectories reach to the switching surface on one side and away from the other side. The state will move around the target operating point. Thus, based on Krein (2001), the conditions leading to rejective mode are given by:

$$\begin{cases} \sigma_{\Delta+}^2|_{\text{on-state}} < 0 \text{ or } \sigma_{\Delta-}^2|_{\text{on-state}} < 0, \dot{\sigma}_{\Delta+}^2|_{\text{on-state}} < 0 \\ \sigma_{\Delta+}^2|_{\text{off-state}} > 0 \text{ or } \sigma_{\Delta-}^2|_{\text{off-state}} > 0, \dot{\sigma}_{\Delta-}^2|_{\text{off-state}} > 0 \end{cases} \quad (8)$$

And the ones for ensuring reflective operation are given by:

$$\begin{cases} \sigma_{\Delta+}^2|_{\text{on-state}} < 0 \text{ or } \sigma_{\Delta-}^2|_{\text{on-state}} < 0, \dot{\sigma}_{\Delta+}^2|_{\text{on-state}} > 0 \\ \sigma_{\Delta+}^2|_{\text{off-state}} > 0 \text{ or } \sigma_{\Delta-}^2|_{\text{off-state}} > 0, \dot{\sigma}_{\Delta-}^2|_{\text{off-state}} < 0 \end{cases} \quad (9)$$

Where ‘on-state’ and ‘off-state’ indicate the corresponding value when the initial state is on state and off state, respectively. For example, $\dot{\sigma}_{\Delta+}^2|_{\text{on-state}}$ is the rate change of $\sigma_{\Delta+}^2$ (ie $\dot{\sigma}_{\Delta+}^2$) when the initial state on and $\dot{\sigma}_{\Delta+}^2|_{\text{off-state}}$ is the rate change of $\sigma_{\Delta+}^2$ (ie $\dot{\sigma}_{\Delta+}^2$) if the initial state is off. According to Eqn (2), Eqn (3) and Eqn (6), if the initial state is on:

$$\sigma_{\Delta+}^2|_{\text{on-state}} = 2(k_{s1} - k_2)(x - x_{\text{ref}})^2 \quad (10a)$$

$$\sigma_{\Delta-}^2|_{\text{on-state}} = 2(k_{s1} - k_1)(x - x_{\text{ref}})^2 \quad (10b)$$

$$\dot{\sigma}_{\Delta+}^2|_{\text{on-state}} = \left. \frac{d\sigma_{\Delta+}^2}{dt} \right|_{\text{on-state}} = 2(k_{s1} - k_2)(x - x_{\text{ref}}) \left. \frac{dx}{dt} \right|_{\text{on}} \quad (x > x_{\text{ref}}) \quad (10c)$$

If the initial state is off:

$$\sigma_{\Delta+}^2|_{\text{off-state}} = 2(k_{s2} - k_2)(x - x_{\text{ref}})^2 \quad (11a)$$

$$\sigma_{\Delta-}^2|_{\text{off-state}} = 2(k_{s2} - k_1)(x - x_{\text{ref}})^2 \quad (11b)$$

$$\dot{\sigma}_{\Delta-}^2|_{\text{off-state}} = \left. \frac{d\sigma_{\Delta-}^2}{dt} \right|_{\text{off-state}} = 2(k_{s2} - k_1)(x - x_{\text{ref}}) \left. \frac{dx}{dt} \right|_{\text{off}} \quad (x < x_{\text{ref}}) \quad (11c)$$

Where $dx/dt|_{\text{on}} > 0$, $dx/dt|_{\text{off}} < 0$ are the slopes of x during on- and off-state, respectively.

By substituting Eqn (10) and Eqn (11) into Eqn (8) and Eqn (9), the regions of the control parameters for rejective mode and reflective mode were given by Eqn (12) and Eqn (13), respectively:

$$k_1 < k_{s2} \text{ and } k_2 > k_{s1} \quad (12)$$

$$k_1 > k_{s1} \text{ and } k_2 < k_{s2} \quad (13)$$

The conditions for refractive mode are within the ones outside the conditions of Eqn (12) and Eqn (13). However, global stability can only be assured if each successive intersection with the switching surface brings the operation closer to the target operating point as illustrated in Fig 6. ‘A’ (x_n, y_n), ‘B’ (x_{n+1}, y_{n+1}) and ‘C’ (x_{n+2}, y_{n+2}) are three subsequent intersections between the state trajectories and the switching surface, called successor points. The condition for stable operation was given by:

$$S_n < S_{n+2} \quad (14)$$

S_n is the distance between ‘A’ and the target point ‘O’ ($x_{\text{ref}}, y_{\text{ref}}$), $S_n^2 = (x_n - x_{\text{ref}})^2 + (y_n - y_{\text{ref}})^2$. S_{n+2} is the distance between ‘C’ and ‘O’, $S_{n+2}^2 = (x_{n+2} - x_{\text{ref}})^2 + (y_{n+2} - y_{\text{ref}})^2$.

By neglecting the hysteresis band $2\Delta y$, for a generic successor point on $\sigma^2 = 0$:

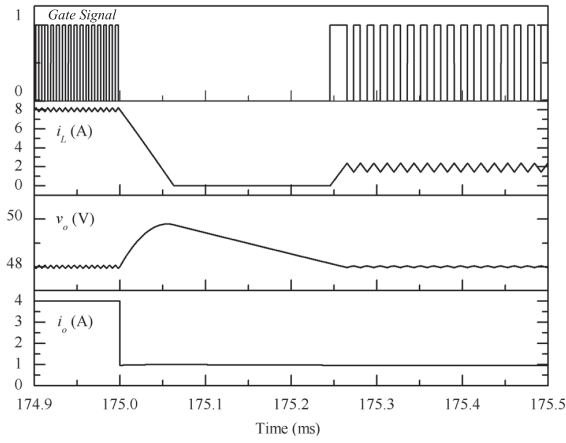
$$y_n - y_{\text{ref}} = k(x_n - x_{\text{ref}})^2 \quad (15)$$

Therefore the condition given in Eqn (14) can be rewritten as:

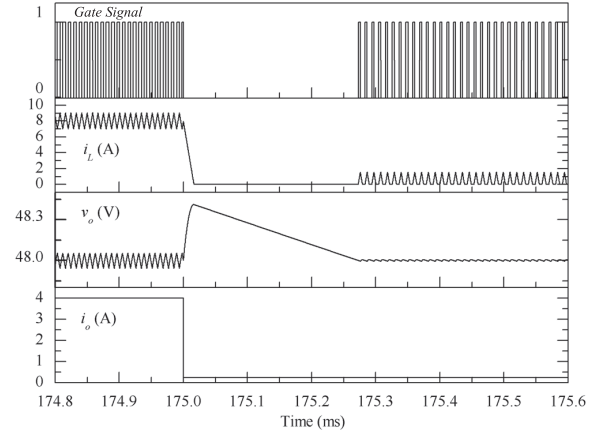
$$(x_{n+2} - x_{\text{ref}})^2 < (x_n - x_{\text{ref}})^2 \quad (16)$$

As shown in Fig 6, the successor points ‘A’ and ‘C’ are on σ_+^2 and ‘B’ is on σ_-^2 . Both of ‘A’ and ‘B’ are on the off-state trajectory while ‘B’ and ‘C’ are on the on-state trajectory. According to Eqn (2), Eqn (3), Eqn (6) and Eqn (15), it can be derived that:

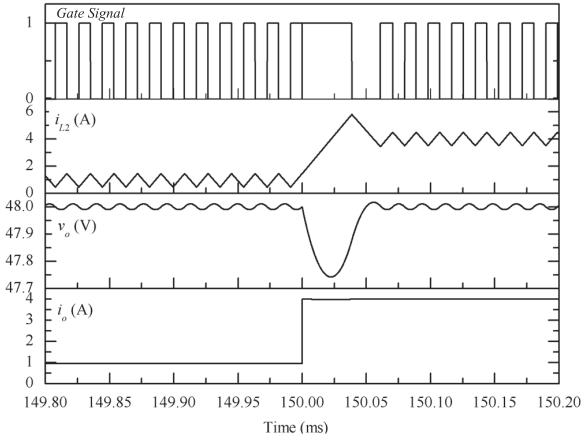
$$(x_{n+2} - x_{\text{ref}})^2 = (x_n - x_{\text{ref}})^2 \frac{k_{s1} - k_1}{k_{s1} - k_2} \frac{k_{s2} - k_2}{k_{s2} - k_1} \quad (17)$$



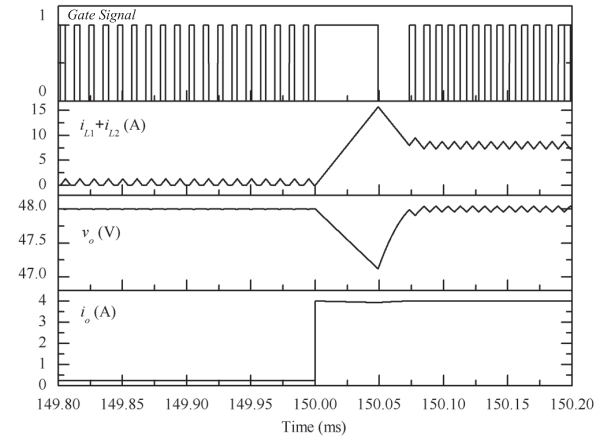
(a) Boost Converter with Load Change from $12\ \Omega$ to $50\ \Omega$ (CCM to CCM) ($V_{in} = 24\text{ V}$, $V_{o,ref} = 48\text{ V}$, $L = 0.2\text{ mH}$, $C = 110\text{ }\mu\text{F}$)



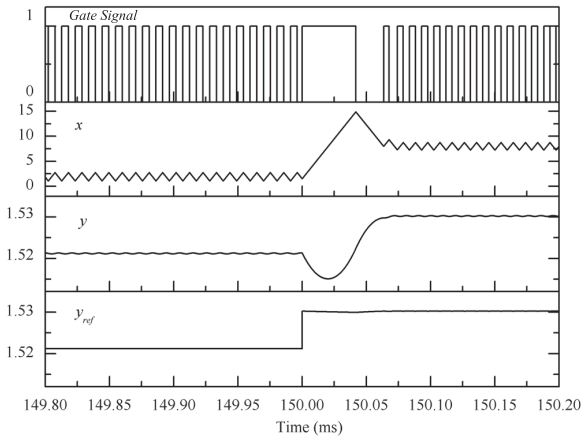
(b) Buck-boost Converter with Load Change from $12\ \Omega$ to $200\ \Omega$ (CCM to DCM) ($V_{in} = 48\text{ V}$, $V_{o,ref} = 48\text{ V}$, $L = 0.1\text{ mH}$, $C = 150\text{ }\mu\text{F}$)



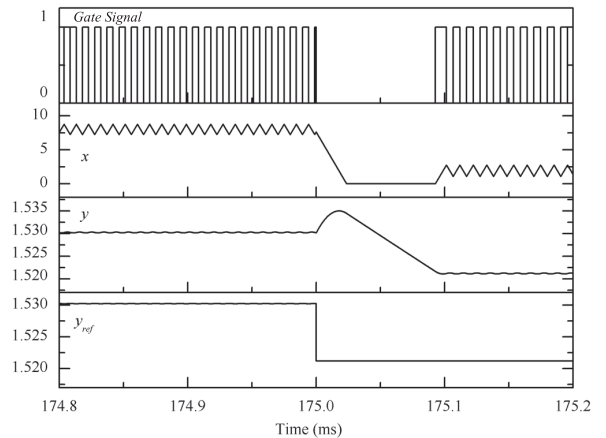
(c) Ćuk Converter with Load Change from $50\ \Omega$ to $12\ \Omega$ (CCM to CCM) ($V_{in} = 48\text{ V}$, $V_{o,ref} = 48\text{ V}$, $L_1 = 1\text{ mH}$, $L_2 = 0.45\text{ mH}$, $C_1 = 220\text{ }\mu\text{F}$, $C_2 = 110\text{ }\mu\text{F}$)



(d) SEPIC Converter with Load Change from $200\ \Omega$ to $12\ \Omega$ (DCM to CCM) ($V_{in} = 48\text{ V}$, $V_{o,ref} = 48\text{ V}$, $L_1 = L_2 = 0.3\text{ mH}$, $C_1 = 4.7\text{ }\mu\text{F}$, $C_2 = 220\text{ }\mu\text{F}$)



(e) SEPIC Converter with Load Change from $50\ \Omega$ to $12\ \Omega$ (x-y Waveforms) ($V_{in} = 48\text{ V}$, $V_{o,ref} = 48\text{ V}$, $L_1 = L_2 = 0.3\text{ mH}$, $C_1 = 4.7\text{ }\mu\text{F}$, $C_2 = 220\text{ }\mu\text{F}$)



(f) SEPIC Converter with Load Change from $12\ \Omega$ to $50\ \Omega$ (x-y Waveforms) ($V_{in} = 48\text{ V}$, $V_{o,ref} = 48\text{ V}$, $L_1 = L_2 = 0.3\text{ mH}$, $C_1 = 4.7\text{ }\mu\text{F}$, $C_2 = 220\text{ }\mu\text{F}$)

Figure 5 – Simulations on Boost Converter, Buck-boost Converter, Ćuk Converter and SEPIC with the Proposed Uniform Control Method

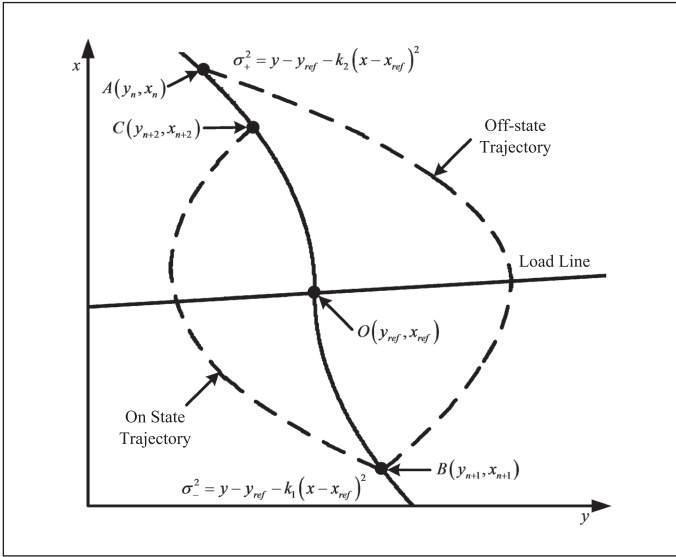


Figure 6 – Illustration of the Movement of Successor in Refractive Mode

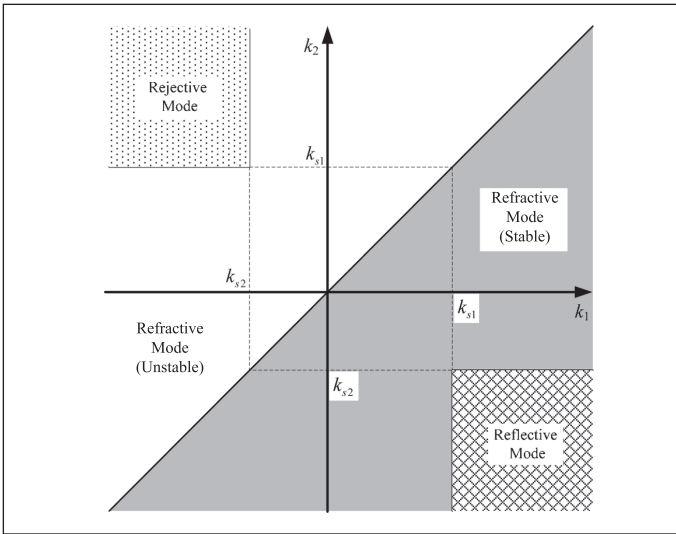


Figure 7 – Stability Analysis of the Proposed Uniform Second-order Switching Surface with Different Control Parameters k_1 and k_2

In order to satisfy Eqn (16), it results in:

$$k_1 > k_2 \quad (18)$$

Fig 7 summarises the aforementioned analysis and presents k_1 and k_2 regions for different operation modes. It can be observed that when $k_1 = k_{s1}$ and $k_2 = k_{s2}$, the switching surface is along the boundary of the reflective and refractive regions. Fig 8 shows five typical switching surfaces with specific values of k_1 and k_2 . 'A' and 'B' are points on the right and left sides of switching surfaces of σ_1^2 , σ_2^2 and σ_3^2 , and 'A' and 'C' locate on the two sides of switching surfaces σ_4^2 and σ_5^2 . σ_{n+}^2 ($n = 1, 2, 3, 4, 5$) and σ_{n-}^2 indicate turn off/on part of one specific switching surface above and below the load line, respectively. It can be noted that the curved switching surfaces σ_1^2 , σ_3^2 , σ_4^2 and σ_5^2 will make arbitrary point operate in reflective, stable refractive, unstable refractive and reflective mode, respectively. When σ_2^2 was selected, which approximately follows the on/off state trajectory, the arbitrary point 'A' or 'B' will reach the target operating point by two switching actions.

Derivation and Implementation Example of Buck-boost Converter

With the aid of Fig 9, the derivation procedures for getting the values

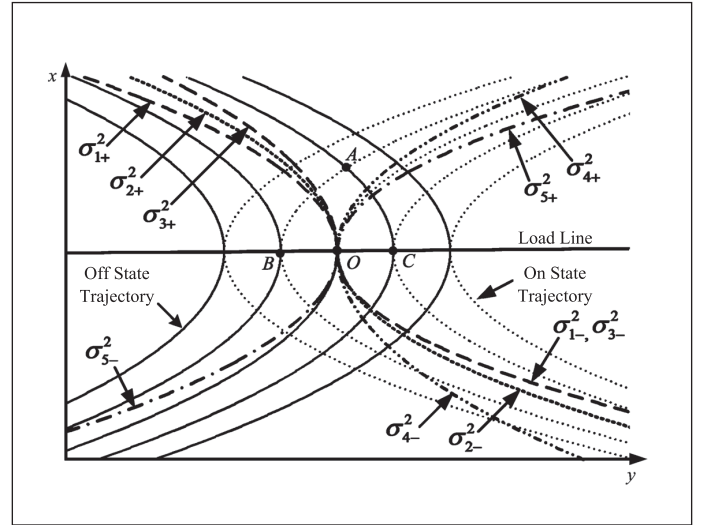


Figure 8 – Illustration of Rejective, Refractive and Reflective Operation Modes

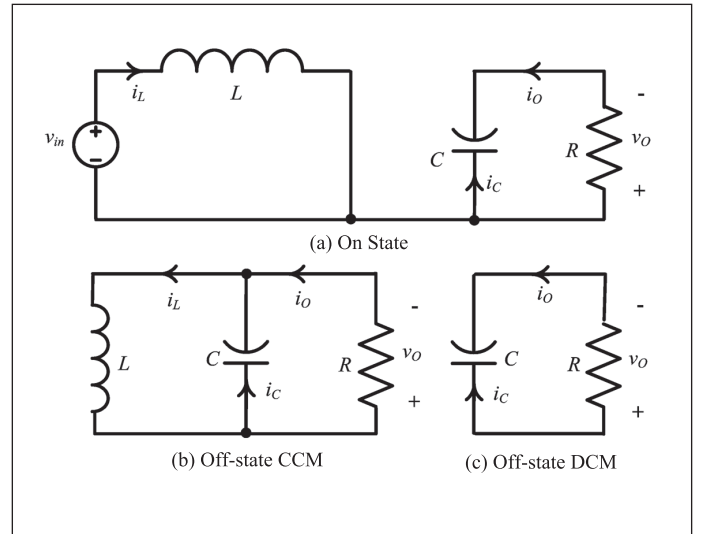


Figure 9 – Operation Modes of Buck-boost Converter

tabulated in Table 1 and 2 are explained by a buck-boost converter as follows:

Step 1 – Derivation of the load line. The load line is derived by considering the average steady state inductor current $i_{L,ref}$. For buck-boost converter:

$$i_{L,ref} = \frac{v_o i_o}{v_{in}} + i_o \quad (19)$$

Where v_{in} is the input voltage, and v_o and i_o are the output voltage and current, respectively.

Step 2 – Determination of x , y and k_{s1} . For buck-boost converter, the control variable x is firstly selected as the inductor current i_L . As shown in Fig 9(a), during on-state period:

$$\frac{dv_o}{di_L} = -\frac{Lv_o}{RCv_{in}} \quad (20)$$

Where L and C are the inductor and output capacitor, respectively, and R is the equivalent load resistance.

By using Eqn (1), it can be shown that:

$$y = 2k_{s1} \int (i_L - i_{L,ref}) di_L = \frac{2k_{s1}}{L} \left(\frac{1}{2} Li_L^2 + \frac{1}{2} Cv_o^2 + Cv_{in}v_o \right) \quad (21)$$

Hence, y can be chosen as:

$$y = \frac{1}{2} Li_L^2 + \frac{1}{2} Cv_o^2 + Cv_{in}v_o \quad (22)$$

$$k_{s1} = \frac{L}{2} \quad (23)$$

Step 3 – Determination of k_{s2} . As shown in Fig 9(b), during off-state period:

$$\frac{dy}{di_L} = \frac{dy}{dt} \frac{dt}{di_L} = -\frac{Lv_{in}}{v_o} (i_L - i_{L,ref}) \quad (24)$$

$$k_{s2} = -\frac{Lv_{in}}{2v_o} \quad (25)$$

Step 4 – Calculation of y_{ref} . Based on Eqn (19) and Eqn (22):

$$y_{ref} = \frac{1}{2} L \left(\frac{v_o i_o}{v_{in}} + i_o \right)^2 + \frac{1}{2} Cv_{o,ref}^2 + Cv_{in}v_{o,ref} \quad (26)$$

Step 5 – Formation of the control law. According to Eqn (7), the switching criterion was described as follows. The switch S is turned on if:

$$y(t) - k_1(i_L - i_{L,ref})^2 - (y_{ref} - \Delta y) \leq 0 \text{ and } i_L(t) < i_{L,ref} \quad (27a)$$

S is turned off if:

$$y(t) - k_2(i_L - i_{L,ref})^2 - (y_{ref} + \Delta y) \geq 0 \text{ and } i_L(t) > i_{L,ref} \quad (27b)$$

An ideal second-order switching surface is that $k_1 = k_{s1}$ and $k_2 = k_{s2}$.

Step 6 – Implementation of control law. Based on the above derivations, the block diagram of the controller for buck-boost converter is shown in Fig 10.

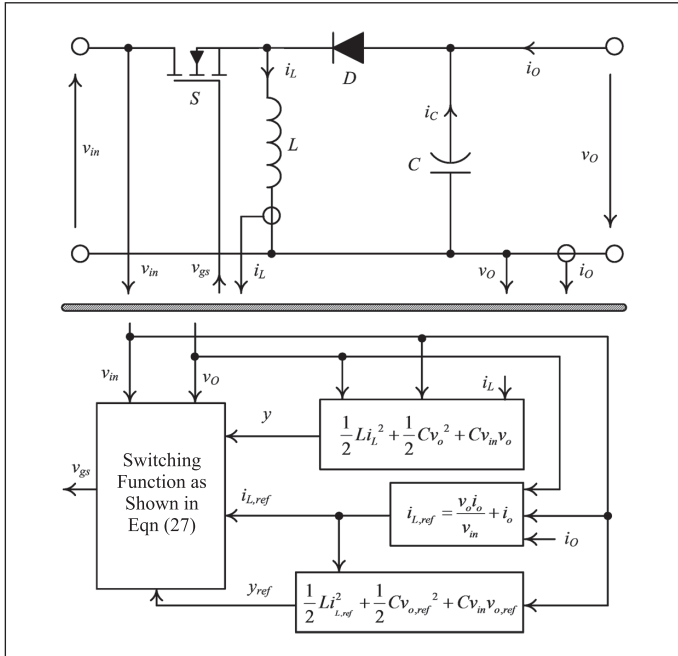


Figure 10 – Implementation of the Proposed Uniform Control Law for Buck-boost Converter

Experimental Verifications

A 190 W, 48/48 V prototype of buck-boost converter was built with an inductor of 0.36 mH and output capacitor of 150 μ F. For the sake of investigation, the proposed control method is implemented according to Fig 10 by a digital method. The prototype was tested under six kinds of load change conditions: a) output resistance is from 47 Ω to 12.3 Ω ,

and vice versa; b) the value of output resistor reduces from 100 Ω to 12.3 Ω , and vice versa, and c) a large load variation with output resistance from 500 Ω to 13.6 Ω , and vice versa.

Figs 11(a) and 12(a) present waveforms of the output voltage v_o , output current i_o , inductor current i_L and switching signal V_{GS} under a load change from 1.02 A to 3.9 A and vice versa. The converter operated in CCM in both loading conditions. It can be observed from the gate signal V_{GS} that the output voltage approaches to the steady-state after two switching actions. The setting time to reach steady state is 230 μ s and 242 μ s, respectively for load increasing and load decreasing.

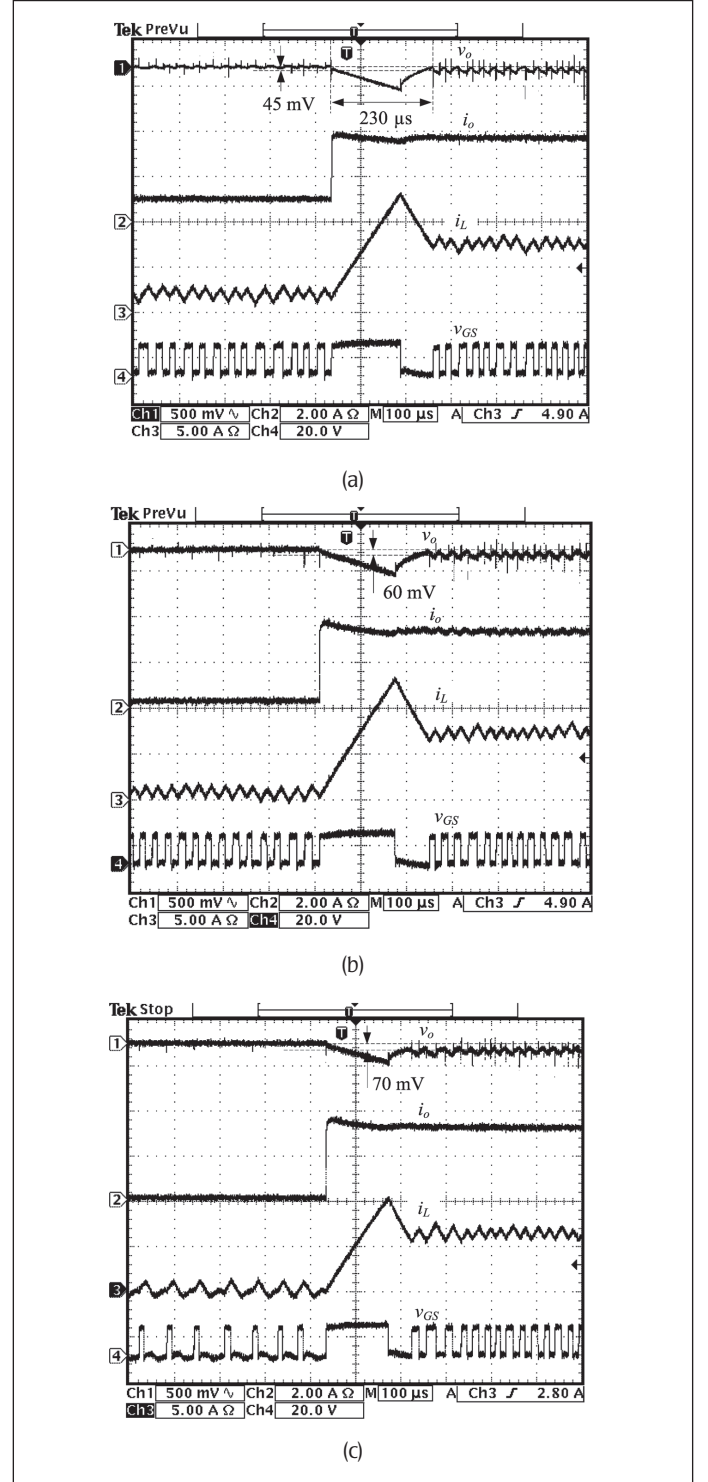


Figure 11 – Waveforms of Dynamic Response under Increased Load Conditions. (a) R_L from 47 Ω to 12.3 Ω , (b) R_L from 100 Ω to 12.3 Ω , (c) R_L from 500 Ω to 13.6 Ω . (v_o : (ac) 500 mV/div, i_o : 2 A/div, i_L : 5 A/div, v_{GS} : 20 V/div and Timebase: 100 μ s/div).

Figs 11(b) and 12(b) show the dynamic response of the converter when the load was switched between the value operating in critical current conduction mode and rated value. The load currents were 0.48 A and 3.9 A, respectively. The inductor current reached to zero at the end of each cycle. A fast dynamic response and well output voltage regulation still can be maintained.

Figs 11(c) and 12(c) give the dynamic response when the operation mode of the converter change from DCM to CCM or versus after large signal disturbance. The load currents were switched between 0.096 A and 3.53 A. It can be observed that the converter reached steady state

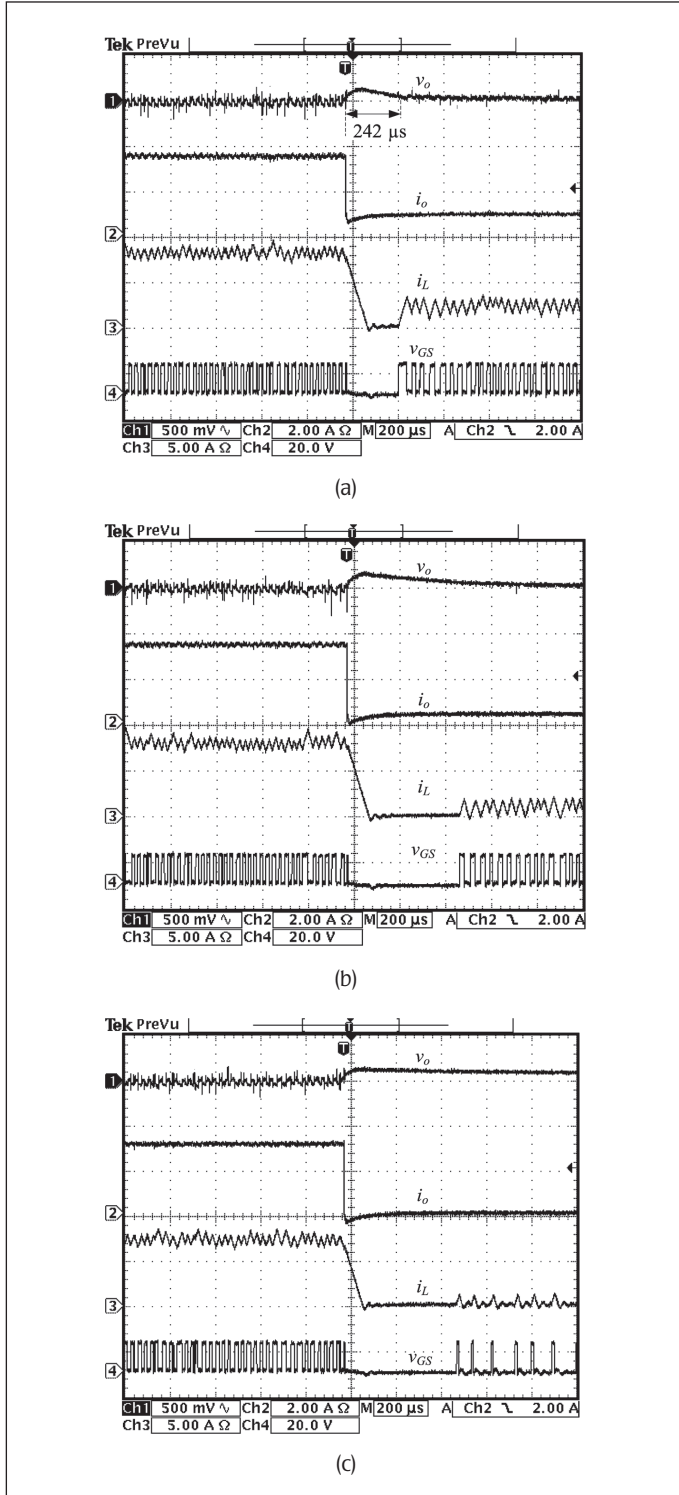


Figure 12 – Waveforms of Dynamic Response under Reduced Load Conditions. (a) R_L from 12.3Ω to 47Ω , (b) R_L from 12.3Ω to 100Ω , (c) R_L from 13.6Ω to 500Ω . (v_o : (ac) 500 mV/div , i_o : 2 A/div , i_L : 5 A/div , v_{GS} : 20 V/div and Timebase: $200 \mu\text{s/div}$).

by two switching actions for energy absorbing and releasing, confirming that the same controller was suitable for both CCM and DCM operation conditions. The variations of average output voltages are 45 mV, 60 mV and 70 mV for the above three kind of load changes, respectively, as indicated in Figs 11(a) to 11(c).

Fig 13 shows the output voltage, output current and driving signal when start-up. The switch is always off to release the energy stored in the inductor to the load to increase the output voltage level before reaching steady-state. The start-up time is measured with 2.15 ms.

Fig 14 presents the measured switching frequency at $R_c = 1.4 \text{ m}\Omega$. With output capacitor of low ESR, the switching frequency is found to vary 4.2 kHz when the converter is operated from critical conduction mode to the rated load.

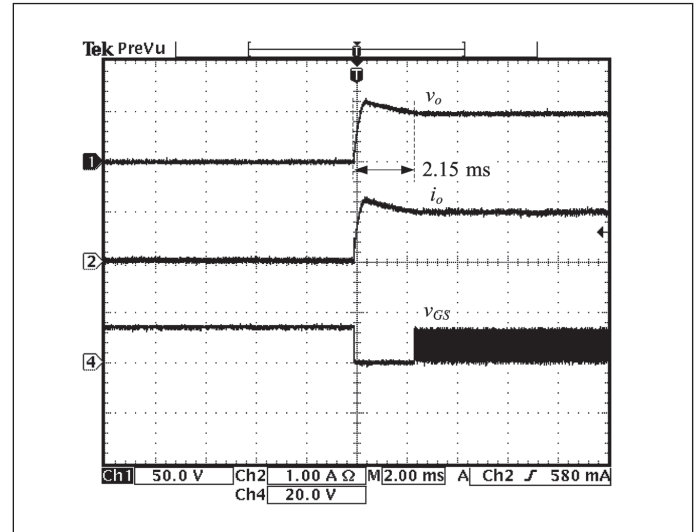


Figure 13 – Waveforms of Start-up Period of Buck-boost Converter (v_o : 50 V/div , i_o : 1 A/div , v_{GS} : 20 V/div and Timebase: 2 ms/div)

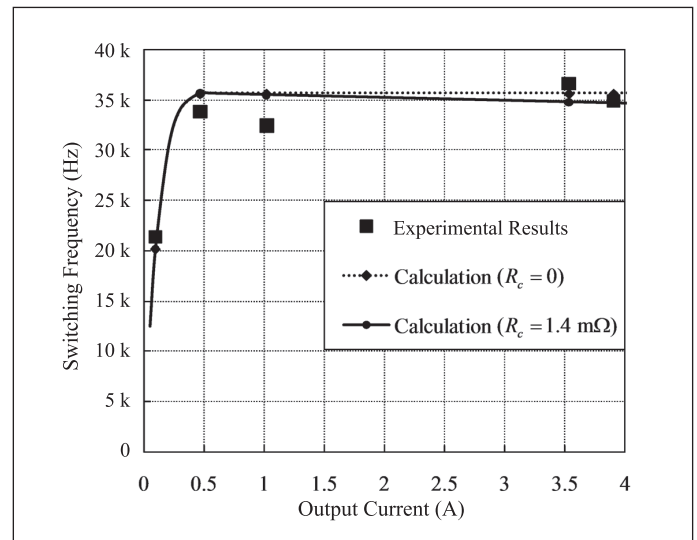


Figure 14 – Measured Switching Frequency of Buck-boost Converter

Conclusion

A unified second-order switching surface is proposed to overcome one of the fundamental limitations of switching surface control, making it possible to apply the same control concept and associated control techniques of buck converter to other basic DC-DC converters. State variables such as capacitor voltages and inductor currents are transformed to Cartesian x-y plane to formulate a general form of switching criterion for different types of DC-DC converters. Accordingly, a uniform control law is formulated, which incorporates the advantages of boundary control with second-order

switching surface for buck converter. The control parameters are obtained readily by considering the component values of the power stage without any sophisticated calculations. Simulation results on boost converter, buck-boost converter, Ćuk converter and SEPIC verify the universal applicability of the proposed uniform controller to basic DC-DC converters. Stability analysis has been carried out by varying the control parameters in different regions. Derivation procedures and design approaches were demonstrated with a buck-boost converter. A 48/48 V 190 W prototype has been built and tested under both CCM and DCM conditions. It has been verified that the control method addresses a complete operation and does not differentiate startup, transient, and steady-state intervals with fast transient response. The results of the large-signal response are in good agreement with the theoretical predictions.

Acknowledgements

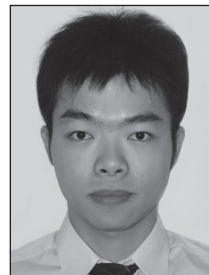
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References

1. Babazadeh A., Maksimović, Hybrid Digital Adaptive Control for Fast Transient Response in Synchronous Buck DC-DC Converters. *IEEE Transactions Power Electron.* Volume 24, No 11, pp2625-2638. Institute of Electrical and Electronics Engineers. USA (2009).
2. Calvente J., Martinez-Salamero L., Valderrama H., Vidal-Idiarte E., Using magnetic coupling to eliminate right half-plane zeros in boost converters. *IEEE Power Electron. Letter.* Volume 2, No 2, pp58-62. Institute of Electrical and Electronics Engineers. USA (2004).
3. Corradini L., Costabeber A., Mattavelli P., Saggini S., Parameter-Independent Time-optimal Digital Control for Point-of-Load Converters. *IEEE Transactions Power Electron.* Volume 24, No 10, pp2235-2248. Institute of Electrical and Electronics Engineers. USA (2009).
4. Chiu J. Y. C., Leung K. S., Chung H. S. H., High-order switching surface in boundary control of inverters. *IEEE Transactions Power Electron.* Volume 22, No 5, pp1753-1765. Institute of Electrical and Electronics Engineers. USA (2007).
5. Erickson R. W., Maksimovic D., *Fundamental of Power Electronics*. Springer. USA (2001).
6. Greuel M., Muyshondt R., Krein P. T., Design approaches to boundary controllers. *Proceeding IEEE Power Electron Special Conference.* pp672-678. Institute of Electrical and Electronics Engineers. USA (1997).
7. Huang S., Xu H., Liu Y., Sliding mode controlled switching regulator with fast response and first-order dynamic characteristic. *Proceeding IEEE Power Electron Special Conference.* pp124-129. Institute of Electrical and Electronics Engineers. USA (1989).
8. Krein P. T., *Elements of Power Electronics*. Chapter 17. Oxford University Press. USA (1998).
9. Krein P. T., *Nonlinear Phenomena in Power Electronics: Attractors, Bifurcation, Chaos, and Nonlinear Control*. Chapter 8. IEEE Press. USA (2001).
10. Krein P. T., Feasibility of Geometric Digital Controls and Augmentation for Ultrafast DC-DC Converter Response. *IEEE COMPEL Workshop.* pp48-56. Institute of Electrical and Electronics Engineers. USA (2006).
11. Leung K. S., Chung H. S. H., Derivation of a second-order switching surface in the boundary control of buck converters. *IEEE Power Electronics Letter.* Volume 2, No 2, pp63-67. Institute of Electrical and Electronics Engineers. USA (2004).
12. Leung K. S., Chung H. S. H., A comparative study of the boundary control of buck converters using first- and second-order switching surfaces-Part I: continuous conduction mode. *Proceeding IEEE Power Electron Special Conference.* pp2133-2139. Institute of Electrical and Electronics Engineers. Brazil (2005).
13. Leung K. S., Chung H. S. H., A comparative study of the boundary control of buck converters using first- and second-order switching surfaces. *IEEE Transactions Power Electron.* Volume 22, No 4, pp1196-1209. Institute of Electrical and Electronics Engineers. USA (2007).
14. Malesani L., Rossetto L., Spiazzi G., Tenti P., Performance Optimisation of Ćuk Converters by Sliding-mode Control. *IEEE Transactions Power Electron.* Volume 10, No 3, pp302-309. Institute of Electrical and Electronics Engineers. USA (1995).
15. Mattavelli P., Rossetto L., Spiazzi G., Tenti P., General-purpose sliding-mode controller for DC-DC converter applications. *Proceeding IEEE Power Electron Special Conference.* pp609-615. Institute of Electrical and Electronics Engineers. USA (1993).

16. Munzert R., Krein P.T., Issues in boundary control. *Proceeding IEEE Power Electron Special Conference.* pp810-816. Institute of Electrical and Electronics Engineers. USA (1996).
17. Redl R., Sun J., Ripple-based Control of Switching Regulators – An Overview. *IEEE Transactions Power Electron.* Volume 24, No 12, pp2669-2680. Institute of Electrical and Electronics Engineers. USA (2009).
18. Song Ting-Ting, Chung H. S. H., Boundary control of boost converters using state-energy plane. *IEEE Transactions Power Electron.* Volume 23, No 2, pp551-563. Institute of Electrical and Electronics Engineers. USA (2008).
19. Spiazzi G., Mattavelli P., *the Power Electronics Handbook*. Chapter 8. CRC Press. USA (2002).
20. Tan S. C., Lai Y. M., Cheung K. H., Tse C. K., On the practical design of a sliding mode voltage controlled buck converter. *IEEE Transactions Power Electron.* Volume 20, No 2, pp425-437. Institute of Electrical and Electronics Engineers. USA (2005).
21. Tan S. C., Lai Y. M., Tse C. K., Cheung K. H., Adaptive feedforward and feedback control schemes for sliding mode controlled power converters. *IEEE Transactions Power Electron.* Volume 21, No 1, pp182-192. Institute of Electrical and Electronics Engineers. USA (2006).
22. Utkin V. I., *Sliding Modes and Their Application in Variable Structure Systems*. Moscow (1974).
23. Utkin V. I., Sabanovic A., Sliding modes applications in power electronics and motion control systems. *Proceeding IEEE International Symposium on Indus Electron.* ppTU22-TU31. Institute of Electrical and Electronics Engineers. USA (1999).
24. Viswanathan K., Oruganti R., Srinivasan D., Dual-mode control of tri-state boost converter for improved performance. *IEEE Transactions Power Electron.* Volume 20, No 4, pp790-797. Institute of Electrical and Electronics Engineers. USA (2005).
25. Wang H., Chung, H. S. H., Presse, J, A Unified Derivation of Second-order Switching Surface for Boundary Control of DC-DC Converters. *Proceeding IEEE Energy Conversion Congress and Exposition.* pp2889-2896. Institute of Electrical and Electronics Engineers. USA (2009).
26. Yan W. T., Chung, H. S. H., Au K., Ho C. N. M., Fixed-frequency boundary control of buck converters with second-order switching surface. *Proceeding IEEE Power Electron. Special Conference.* pp629-635. Institute of Electrical and Electronics Engineers. USA (2008).

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