A NEW ADAPTIVE AIR HANDLING SYSTEM CONTROLLER

Albert T P So*, Bsc(Eng), MPhil, CEng, MHKIE, MCIBSE, MIEE
W L Chan*, Bsc(Eng), MPhil, CEng, MIEE, MIEEE
T T Chow*, BSc(Eng), MSc, MBA, CEng, MHKIE, MCIBSE, MIMechE, MASHRAE
W L Tse+, BEng, GMHKIE, AMIEE

Modern heating, ventilation and air-conditioning (HVAC) systems commonly employ the concept of "Central All-Air System" utilising Air Handling Units (AHU's) and Variable Air Volume (VAV) boxes for energy conservation. Thus, a good controller for the AHU's is extremely desirable.

In this paper, a simulation model for a standard air-handling system is presented. A new controller based on the modern adaptive control algorithms has been developed where the input and actuating variables are incorporated into a system identification model which can predict the new system status based on past records and suggest the optimal control actions. Its performance has been shown to be superior to that of a conventional proportional-integral-differential (PID) controller in at least three aspects: adaptation to system change, response rate and energy conservation.

Keywords: *Adaptive control, system identification, HVAC control, system modelling*

Introduction

The air-handling plant of a large building must be designed to cope with a wide range of operating conditions since the weather and occupants activities are subject to significant, periodic changes from day to night and from season to season. The air-conditioning process is highly non-linear; the interaction between the temperature and humidity control loops is significant and the constraints imposed by non-ideal actuator behaviour are considerable. Conventionally, a cascaded, multi-loop PID control structure has been used. Brandt & Shavit (1984) simulated the response of a PID-controlled discharge air temperature control system to a step change input. In fact, PID control has been considered a successful implementation in air-conditioning control since most practical systems available nowadays are based on this classical approach. However, as seen later, PID control is favourable on the assumption that the system model parameters do not change much. When a well-tuned PID controller is applied to another system with different model parameters, the response becomes poor and energy consumption increases.

Recently, research work has been done on the introduction of adaptive and self-tuning controls (Dexter & Jota, 1985; John & Dexter, 1987; Jota & Dexter, 1986; Jota & Dexter, 1988). However, there are difficulties with such self-tuning controllers, as mentioned by Jota & Dexter (1988).

The major problem of successfully applying the self-tuning controllers is to ensure the parameter estimator functions correctly and produces reasonable estimates of the parameters. Estimates may be particularly unreliable when the valve is operating near to its limits and also when the behaviour of the process is varying rapidly with time.

It has also been found that reliable control of the heater and cooling coil, over their full operating range, is only possible if the self-tuning controllers are detuned. Though the self-tuning controllers are more reliable when cascaded with simple fixed parameter controllers, the design of the inner-loop controllers raises considerable problems when attempts

* *City Polytechnic of Hong Kong.*
\+ *China Light & Power Co Ltd.*

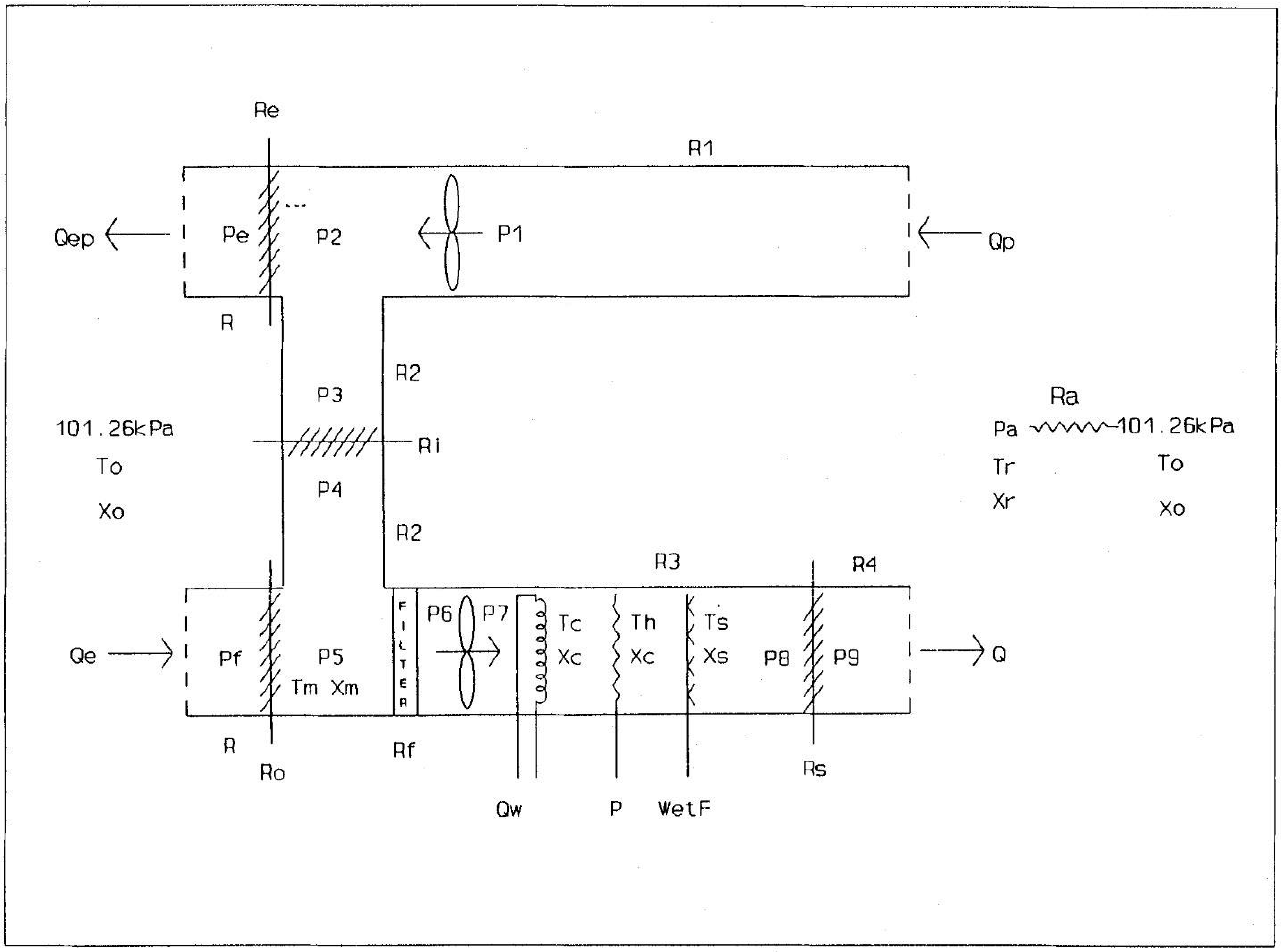

Figure 1: The air handling system model.

are made to improve the control performance by more exactly compensating for the non-linearities of the actuators and valves.

It is our belief that the existing self-tuning controllers are too complicated or the designer is too ambitious to design a unique comprehensive controller for the whole air handling system, resulting in unsatisfactory behaviour. The major problem behind is that the physical relationship between the control parameters and the actuating signals are totally ignored for the existing self-tuning controllers and very often, the actual relationship between certain variables within the control system is very indirect but they are forced together in one estimation equation.

For example, the control becomes highly unstable and very sensitive to external disturbance if the room temperature is related to the operation of all dampers, the chilled water flow rate, the heater, the humidifier and all fans etc. Therefore, we shall follow the conventional design concept for PID controller that keeps the input and output points of each controller to a minimum, trying to produce multi-input and single-output (MISO) controllers within a multi-loop control scheme. In fact, this is equivalent to introducing certain amount of expertise knowledge into the control scheme instead of integrating everything together blindly. System identification is then employed to ensure that the controllers are adaptable to any ad hoc changes in the controlled environment and thus it is also called adaptive control. Computer simulation reveals that such scheme can produce a more stable and reliable control environment compared with that of the existing controllers.

The air handling system model

The air-handling system model used in our study is shown in Figure 1. Here, the thermal condition of the supply air is monitored by a supervisory controller in accordance with the pre-set air temperature and relative humidity levels of the conditioned space. The approach is known as the dew-point control scheme. Return air from the conditioned space enters the return air duct with the flow boostered by the variable speed return air fan. At the AHU inlet, most of the return air (the remaining being exhausted through the exhaust air duct) mixes with fresh outdoor air, in a controlled proportion by adjusting the exhaust and fresh air dampers. The damper of the mixing air chamber is interlocked with both the exhaust and fresh air dampers. The mixed air then passes through the filter and arrives at the chilled water cooling coil where both the air temperature and humidity ratio are reduced. Accurate control of humidity is accomplished by the selective operation of either the reheater or the humidifier. The supply air flow rate is controlled by the supply air damper.

The aim of the control action on the whole is to achieve the desired room temperature and humidity levels with minimum delay time and energy expenses. Winter heating is ignored in this study as the consequences will then be quite similar within our focus. A computer programme written in Turbo C++ (an object oriented programming language) has been prepared for simulating this model. The various system components are described using nonlinear algebraic equations and ordinary differential equations. The Euler method is used to numerically integrate the differential equations and at each step, the algebraic equations are solved simultaneously. The various air flow paths within the system are described by a resistance network. All air flow resistances are assumed constant and determined by the system sizing and equipment selection. The characteristics of the fans are modelled by fitting a second order polynomial to the manufacturer's data. The outdoor pressure is taken as a boundary condition and the system flow rates are determined at each time step using the Hardy Cross method.

The symbol list is in Appendix A; the plant data and equations are summarised in Appendix B. This framework is sufficient for testing any new control algorithms designed for the optimal operation of a practical air-handling system interacting with realistic outdoor conditions and room space loads at the transient domain.

Conventional VAV control by PID loops

The conventional air-handling systems can be classified into two major categories (Carrier 1972), namely constant volume-variable temperature and variable volume-constant temperature respectively. The former is designed for both temperature and humidity control where necessary while the latter has particular merits on energy efficiency but the humidity control may not be too satisfactory. VAV control, a concept well adopted in Hong Kong, is chosen to test the two controllers, i.e. PID and system identification (or adaptive). The physical constraints of the actuators are listed in Appendix C.1. A single zone control environment is employed with reference to Figure 1. The speed of supply air fan is controlled by a variable voltage variable frequency (VVVF) motor drive to fix the pressure behind the supply air damper (P8)to a constant value (P8s). The supply air damper (Rs) is controlled by a stepper motor to maintain the room air temperature at a desired value (Trs). The heater and the humidifier are controlled to maintain the room relative humidity at a desired value (RHs). The chilled water flow rated is so controlled by a regulating valve to maintain the supply air temperature at a constant value (Tss). The other dampers and the speed of the return air fan are coordinated to maintain the fresh air supply rate or the equivalent exhaust air flow rate at a desired value. PID control is used where the deviation e(k) of the control parameter from the set point at the "kth" time step is used to control the actuating command ac(k+1) at "(k+1)th" time step according to:

$$
\begin{aligned}
\Delta ac(k) &= K_p\, e(k) + K_d\,[\, e(k) - e(k-1)\,] + K_i \sum_{i=1}^{k} e(i) \\
ac(k+1) &= ac(k) + \Delta ac(k)
\end{aligned}
\qquad (1)
$$

where ac = *actuating command*
K_p = *proportional gain*
K_d = *differential gain*
K_i = *integral gain*
e = *error or deviation from setpoint*

Control by system identification

An operational model of a system is used to estimate the change in any parameter based on the past records. The model is actually represented by a certain number of unknown parameters and the job is to find out all these unknown parameters, i.e. fixing the model, based on the continuous operation of the system and data acquisition. Once the model has been fixed, the status in future can be estimated due to the current control actions. This is the basic concept of system control by system identification and there are good references available (Astrom, 1983; Wellstead, 1991). This involves solving a set of linear equations with the number of equations highly exceeding the number of variables. However, the model parameters change as the operation continues and therefore, we have to keep track of such changes so that the updated model of the system is known and the control becomes effective.

Principle of system identification

A typical model that is applicable to our case is shown in Figure 2 where the following equation clearly defines the system:

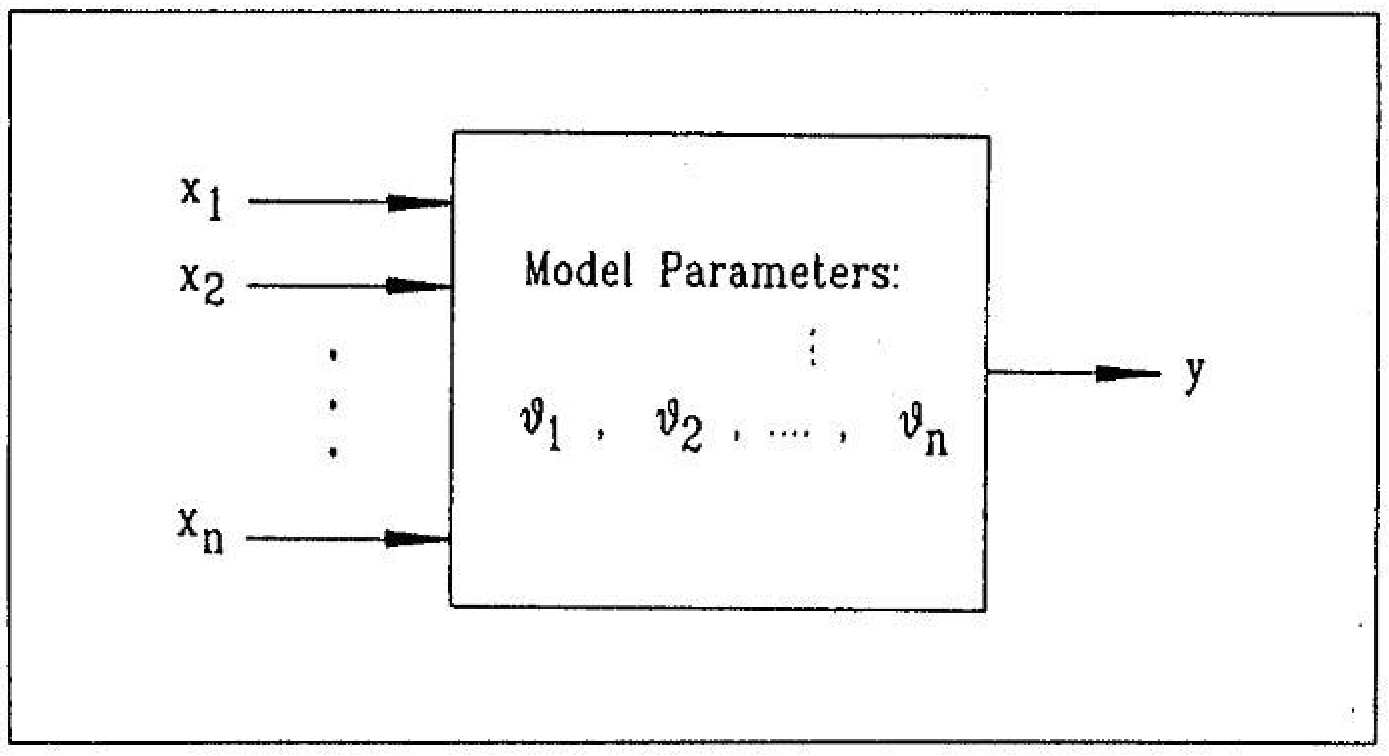

Figure 2: Model basis of system identification.

$$ y = \sum_{i=1}^{n} \theta_i x_i \qquad (2) $$

where θ_i's are the unknown parameters that need to be determined. x_i's can be anything relevant and it can be y itself. It is assumed that we continuously monitor x's and y at discrete time step, at t = 1, t = 2, until t = m and y(k) and $x_i(k)$'s are used to denote the monitored values at the kth time step, where i = 1 ,..., n and k = 1 ,...., m. Thus the following m number of equations shall describe the whole system.

$$y(k) = \sum_{i=1}^{n} \theta_i x_i(k) \qquad for\ k = 1, ..., m \tag{3}$$

For the purpose of control by system identification, $x_1(k)$ can be equal to y(k-1) and $x_3(k)$ can be equal to $x_2(k-1)$ so that the system can be identified by tracing the past records, e.g. for the control of supply air fan as shown in Appendix D, y(k) = P8(k); $x_1(k)$ = P8(k-1) and $x_2(k)$ = Ns(k) so that n = 2. Equation (3) can be written in the matrix form $\mathbf{y} = \mathbf{X}\theta$. In order to find out the n number of unknown parameters, $\theta_1, \ldots, \theta_n$, m should be larger than or equal to n. If m = n, there will be an unique solution. However, there are intrinsic errors within the system with respect to the monitoring and simulation procedures. Thus, m is chosen to be larger than n and an error vector, $(\varepsilon_1, \ldots, \varepsilon_m)^T$ is introduced for the m number of equations so that

$$\begin{aligned} &\varepsilon = y - X\theta \\ &\textit{The job is to choose a } \hat{\theta} \textit{ such that} \\ &J = \sum_{k=1}^{m} \varepsilon_k^2 = \varepsilon^T \varepsilon \quad \textit{is minimal} \end{aligned} \tag{4}$$

$$\begin{aligned} J &= (y - X\theta)^T (y - X\theta) \\ &= y^T y - \theta^T X^T y - y^T X\theta + \theta^T X^T X\theta \\ \left.\frac{\partial J}{\partial \theta}\right|_{\theta=\hat{\theta}} &= -2X^T y + 2X^T X\hat{\theta} = 0 \\ \textit{This implies} \quad X^T X\hat{\theta} &= X^T y \qquad or \quad \hat{\theta} = (X^T X)^{-1} X^T y \end{aligned} \tag{5}$$

This procedure gives an optimal solution when m number of sampled data sets are available. However, the system parameters change continuously and therefore we have to keep track of such change. In other words, whenever a new set (m+1) of sampled data is available, the optimal solution has to be computed again using the latest m number of equations, which is very time consuming and computationally intensive. At this point, there are (m+1) sets of sampled data, as shown below:

$$y_{m+1} = \begin{bmatrix} y(1) \\ \vdots \\ y(m) \\ \cdots\cdots \\ y(m+1) \end{bmatrix} = \begin{bmatrix} y_m \\ \cdots\cdots \\ y(m+1) \end{bmatrix} \qquad X_{m+1} = \begin{bmatrix} x_1(1) & \cdots\cdots & x_n(1) \\ \vdots & & \vdots \\ x_1(m) & \cdots\cdots & x_n(m) \\ \cdots\cdots & \cdots\cdots & \cdots\cdots \\ x_1(m+1) & \cdots\cdots & x_n(m+1) \end{bmatrix} = \begin{bmatrix} X_m \\ \cdots\cdots \\ x^T(m+1) \end{bmatrix} \tag{6}$$

Therefore, the method of recursive least squares (Young 1984) with the introduction of forgetting factor is employed to reduce the computational burden. The forgetting factor, λ, is introduced in the minimisation of the total weighted squared error (J_m) when m sets of sampled data are available.

$$J_m = \sum_{k=1}^{m} \lambda^{m-k} \varepsilon(k)^2 \qquad 0 < \lambda < 1 \tag{7}$$

The recursive least squares solution can be obtained as:

$$\hat{\theta}(m+1) = \hat{\theta}(m) + \Gamma(m+1)\left[\, y(m+1) - x^T(m+1)\hat{\theta}(m) \,\right]$$

$$where \quad \begin{cases} \Gamma(m+1) = \dfrac{1}{\lambda + x^T(m+1)P(m)x(m+1)} P(m)\, x(m+1) \\ P(m+1) = \dfrac{1}{\lambda}\left[\, P(m) - \Gamma(m+1)x^T(m+1)P(m) \,\right] \\ P(m) = \dfrac{1}{\lambda}\left(X_m^T X_m \right)^{-1} \end{cases} \tag{8}$$

λ is set to 0.9 in our case. When λ is equal to one, the solution becomes the ordinary least squares solution. In general, the estimation is more influenced by the more recently obtained data sets. There is obviously a trade-off between tracking ability and noise sensitivity. It is quite impossible to accurately follow parameters which change fast but a slow time variation can often be tracked reasonably well as in our case. In this way, provided that the optimal solution at the "mth" time step is available, the optimal solution at the "(m+1)th" time step can be easily obtained by equation (8) without the need to solve a large set of simultaneous equations again.

The control algorithm

At the mth time step, it is assumed that the optimal solution is available, or in the other words,

$$y(m) \approx \sum_{i=1}^{n} \hat{\theta}_i x_i(m) \tag{9}$$

If one of the x_i's , namely the x_c , is related to the actuating command. It is possible to calculate the actuating command ac(m+1) required for the "(m+1)th" time step by solving the equation (10).

$$\begin{aligned} y_s &= \left\{ \sum_{i=1, i\neq c}^{n} \hat{\theta}_i x_i(m) \right\} + \hat{\theta}_c x_c(m+1) \\ \rightarrow \quad x_c(m+1) &= \frac{y_s - \left\{ \sum_{i=1, i\neq c}^{n} \hat{\theta}_i x_i(m) \right\}}{\hat{\theta}_c} = \Delta ac \\ \rightarrow \quad ac &= ac + \Delta ac \\ where \quad y_s &= \textit{desirable set point of } y \end{aligned} \tag{10}$$

Of course, if there are two or more variables that are related to the actuating commands, the equations for time step (m+1) and (m+2) are required and y(m+1) = y(m+2) = y_s.

This will increase the amount of error associated with the system identification procedure and this fully justifies our task to use simple and single controller within a multi-loop control scheme.

General discussion on computer simulations

The simplified AHU model described earlier is used as a basis for studying the performance of the two types of controller discussed so far, namely PID based control and system identification based control. A single zone served by the AHU is chosen for the simulation exercise. The initial room temperature is 33°C and the initial room relative humidity is 70%. The initial desired room temperature setting is 22°C and the associated relative humidity setting is 60%. After 2000 seconds, the room temperature setting is raised to 24°C but the relative humidity setting remains unchanged. Throughout this operational duration, we can understand the performance of the two controllers with respect to their stability, the response rate and the total energy consumption.

Performance of PID-based AHU controllers

First of all, the control is executed by a set of well-tuned PID controllers whose settings are shown in Appendix C.2. Figure 3(a) shows the room temperature profile for a period of 4000 seconds after the system is switched on; Figure 3(b) shows the relative humidity profile for the same period; Figure 3(c) shows the real time power consumption of the chilled water cooling coil; Figure 3(d) shows the real time power consumption of the reheater while Figure 3(e) is for the two fans. As a matter for comparison, the optimal settings are slightly changed, as shown in Appendix C.3 so that all the PID controllers are detuned. The corresponding performance is shown in Figure 4(a), 4(b), 4(c), 4(d) and 4(e) respectively.

Performance of system identification based controllers

The control is then switched over to a new set of controllers based on system identification, as shown in Appendix D. The input points, output points of each controller and the operational environment remain unchanged during the 4000 second period of simulation. The corresponding results are shown in Figure 5(a), 5(b), 5(c), 5(d) and 5(e).

Remarks of comparison

It can be seen that a well-tuned set of PID controllers can give satisfactory performance. Steady state can be achieved within 300 seconds upon initialisation and 700 seconds upon disturbance for room temperature while that for relative humidity is 800 seconds and 1000 seconds respectively. However, when the PID controllers are slightly detuned, it can be seen that the response rate becomes quite poor and the steady state seems to be never achieved. At the same time, the power consumption increases drastically. By using adaptive control or system identification based control, the response rate is a little bit faster than that of a well-tuned PID system since it does not involve the tuning of any parameters. Experience of an operational expert is used to choose the relevant parameters and the relationship between them, i.e. the format of the model. The actual values of the parameters can be adjusted automatically during the continuous operation of the system. The stability of the system is also enhanced. The power consumptions quoted in the figures refer to the rate of thermal/dynamic energy either delivered to (say heating and fan power) or extracted from (say cooling) the moist air stream. With an estimation of the coefficient of performance of the central chiller plant and the fan/pump efficiency, the total electrical power consumption of the air-handling system within the 4000 seconds of simulation period for each case can be determined. It has been found that the total power consumption for the case of adaptive control is only 87% of the well-tuned PID case. The major saving comes from the reduced reheater operating capacity. Ironically, the total power consumption for the detuned PID case is only 84% of the well tuned case, i.e. the "most economical solution" is actually the worst solution. In fact, the apparent saving is due to the fact that the desirable control target has not been achieved by the detuned PID system.

Conclusions

It is shown in this paper that a well tuned PID set of controllers is, of course, a satisfactory solution for a particular AHU. However, any offset of the settings from the optimal values, say due to human error or time-drifting, will heavily deteriorate the control actions. Actually, during on-site application, it is very very difficult, though not totally impossible, to obtain the optimal settings. Even for our simplified model, extensive effort is required to arrive at the optimal settings listed in Appendix C.2. Therefore, adaptive or system identification based control seems to be an ultimate solution to the problem. It depends on the experience of an expert to fix the format of the model and the details can be adjusted automatically during the course of operation. It does not rely on any correct or optimal tuning of parameters. The existing intrinsic problem of instability associated with self-tuning control that aims at a very comprehensive mode of estimation is removed by introducing simple MISO controllers within a multi-loop control scheme. Such concept must be encouraged for all AHU control and perhaps it can be further extended to chiller control and the whole air-conditioning system.

Acknowledgement

The research project is financially supported by the Strategic Research Grant #700209 of City Polytechnic of Hong Kong.

References

1. Brandt, S. G. & Shavit, G. (1984) Simulations of the PID algorithm for direct digital control application, *Workshop on HVAC Controls, Modelling and Simulation*, Gerorgia Institute of Technology, Atlanta, GA.

2. Dexter, A.L. & Jota, F.G. (1985) Multi-loop self-tuning control of an air-conditioning plant", *Proceedings of the International Symposium on Recent Advances in Control and Operation of Building HVAC Systems*, Trondheim, Norway, pp. 113-123.

3. Jota, F.G. & Dexter, A.L. (1986) Self-tuning control of a chilled water cooling-coil: simulated and experimental assessment, *Proceedings of Second International Conference on System Simulation in Buildings*, Liege, Belgium, Commission of European Communities, pp. 577-599.

4. John, R.W. & Dexter, A.L. (1987) Intelligent buildings: adaptive and self-tuning controls, *Proceedings of the CIBSE Symposium on Advances in Air-conditioning*, Chameleon Press Ltd., London, pp. 107-120.

5. Jota, F.G. & Dexter, A.L. (1988) Self-tuning control of an air-handling plant, *Proceedings of International Conference on Control 88*, Institution of Electrical Engineers, pp. 41-46.

6. Carrier Air-Conditioning Company (1972) *Handbook of Air-Conditioning System Design.*

7. Wellstead, P.E. & Zarrop, M.V. Eds. (1991) *Self Tuning Systems - Control and Signal Processing*, John Wiley & Sons, England, pp. 71-114.

8. Astrom, K. J. (1983) Theory and applications of adaptive control - a survey, *Automatica*, vol. 19, no.5, pp. 471-486.

9. Young, P. Eds. (1984) *Recursive Estimation and Time Series Analysis*, Berlin: Springer-Verlag, pp. 57-63.

Appendix A: MEANING OF SYMBOLS

TimeStep	: simulation time step in sec
rooms	: room sensible load in kW
rooml	: room latent load in kW
roomv	: room volume in m^3
Trs	: room desired air temperature in °C
Tr	: room actual air temperature in °C
To	: outside air temperature in °C
Tm	: mixed air temperature in °C
Tc	: air temperature after cooling coil in °C
Th	: air temperature after reheater in °C
Tcwb	: wet bulb air temperature at humidifier °C
Tss	: supply air desired temperature in °C
Ts	: supply air actual temperature in °C
Twi	: chilled water temperature entering cooling coil in °C
Twm	: average temperature of water in cooling coil in °C
Xr	: room humidity ratio in kg/kg
Xo	: outside humidity ratio in kg/kg
Xm	: mixed air humidity ratio in kg/kg
Xw	: humidity ratio of saturated air at mean cooling coil temperature in kg/kg
Xc	: humidity ratio after cooling coil in kg/kg
Xcwb	: humidity ratio of saturated air at humidifier in kg/kg
Xs	: humidity ratio of supply air in kg/kg
RHs	: room desired relative humidity in %
RH	: room actual relative humidity in %
Nr	: speed of return air fan in rev/min
Ns	: speed of supply air fan in rev/min
Qw	: instantaneous chilled water flow rate at cooling coil in m^3/s
alpha	: portion of saturation air at cooling coil
P	: instantaneous power of reheater in kW
WetF	: portion of air getting saturation at humidifier
Q, Qe	: air flow rate refer to Figure 1 in m^3/s
Qep, Qp	: air flow rate refer to Figure 1 in m^3/s
P8s	: desired pressure before supply air damper in kPa
k	: partial pressure of water vapour at dewpoint temperature after cooling coil in kPa
P1, P2, P3, P4, P5, P6, P7, P8, P9, Pa, Pe, Pf	: total pressure at positions referred to Figure 1 in kPa
R1, R2, R3, R4, Re, R, Ro, Rf, Rs, Ri, Ra	: damper resistances referred to Figure 1 in $kPa/(m^3/s)^2$
Dw	: water density in kg/m^3
Da	: moist air density in kg/m^3
Cpw	: specific heat capacity of water in kJ/(°C kg)
Cpa	: specific heat capacity of moist air in kJ/(°C kg)
Cvw	: latent heat of water in kJ/kg
Pg(T)	: water vapour pressure (at temperature T °C) in kPa
fan(N)_a	: coefficient "a" of a fan at speed N
fan(N)_b	: coefficient "b" of a fan at speed N

Appendix B

1. PLANT DATA

TimeStep	= 1
rooms	= 12.4
rooml	= 4.0
roomv	= 566
Tr	= 33 (initial)
Trs	= 24
To	= 33
Tss	= 14
Twi	= 7
Xr	= 0.0247 (initial)
Xo	= 0.0247
RHs	= 60
Nr	= 200
Ns	= 1000 (initial)
Qw	= 1.4×10^{-3} (initial)
P	= 0 (initial)
alpha	= 0.9
WetF	= 0 (initial)
P8s	= 102.66
R1	= 2.56×10^{-2}
R2	= 3.093×10^{-3}
R3	= 2.32×10^{-2}
R4	= 1.55×10^{-2}
Re	= 1.86
R	= 3.093×10^{-3}
Ro	= 1.86×10^{-2}
Rf	= 1.16×10^{-2}
Rs	= 3.093×10^{-3} (initial)
Ri	= 7.27×10^{-2}
Ra	= 1.08
Dw	= 1000
Cpw	= 4.2
Da	= 1.2
Cpa	= 1.02
Cvw	= 2.26×10^{3}

2. AIR FLOW EQUATIONS

$Pe = 101.26 + R \times Qep^2$
$P2 = Pe + Re \times Qep^2$
$P1 = P2 - 6.89 \times (fan\_a(Nr) - fan\_b(Nr) \times Qp^2 / 2.23e\text{-}7)$
$Pa = 101.26 + Ra \times (Q\text{-}Qp)^2$
$P3 = P2 - R2 \times (Qp\text{-}Qep)^2$
$P4 = P3 - Ri \times (Qp\text{-}Qep)^2$
$Pf = 101.26 - R \times Qe^2$
$P5 = Pf - Ro \times Qe^2$
$P6 = P5 - Rf \times Q^2$
$P7 = P6 + 6.89 \times (fan\_a(Ns) - fan\_b(Ns) \times Q^2 / 2.23e\text{-}7)$
$P9 = Pa + R4 \times Q^2$

$P8 = P9 + Rs \times Q^2$

The above equations can be grouped into 4 independent equations in term of Q, Qp, Qe, Qep. Then, the 4 equations can be solved for Q, Qp, Qe, and Qep using Gauss Elimination.

$(Re + R) \times Qep^2 + R1 \times Qp^2 - Ra \times (Q\text{-}Qp)^2 - 6.89 \times (fan\_a(Nr) - fan\_b(Nr) \times Qp^2 / 2.23e\text{-}7) = 0$
$(R3 + Rs + R4 + Rf) \times Q^2 + (Ro + R) \times Qe^2 + Ra \times (Q\text{-}Qp)^2 -$
$\quad 6.89 \times (fan\_a(Ns) - fan\_b(Ns) \times Q^2 / 2.23e\text{-}7) = 0$
$(Re + R) \times Qep^2 + (R + Ro) \times Qe^2 - (2 * R2 + Ri) \times (Qp\text{-}Qep)^2 = 0$
$Qe + Qp - Q - Qep = 0$

3. MIXING TEMPERATURE AND HUMIDITY

$Tm = ((Qp - Qep) \times Tr + Qe \times To)/Q$
$Xm = ((Qp - Qep) \times Xr + Qe \times Xo)/Q$

4. COOLING COIL

$2 \times Qw \times Dw \times Cpw \times (Twm\text{-}Twi) - Q \times Da \times alpha \times (Cpa \times (Tm\text{-}Twm) + Cvw \times (Xm\text{-}Xw)) = 0$
gives Twm
$Xw = 0.622 \times Pg(Twm) / (P8 - Pg(Twm))$
$Tc = alpha \times Twm + (1 - alpha) \times Tm$
$Xc = alpha \times Xw + (1 - alpha) \times Xm$

5. REHEATER

$Th = Tc + P/(Q \times Da \times Cpa)$
Humidity ratio remains unchanged.

6. HUMIDIFIER

$k - Pg(x) + (P8 - Pg(Tcwb)) \times (Th - Tcwb)/(1546.62 - 1.44 \times Tcwb) = 0$ gives Tcwb
$Xcwb = 0.622 \times Pg(Tcwb) / (P8 - Pg(Tcwb))$
$Ts = (1 - WetF) \times Th + WetF \times Tcwb$
$Xs = (1 - WetF) \times Xc + WetF \times Xcwb$

7. ROOM TEMPERATURE AND HUMIDITY

$Tr_{new} = Tr_{old} + TimeStep \times (rooms/(Da \times Cpa) - Q \times (Tr\text{-}Ts)) / roomv$
$Xr_{new} = Xr_{old} + TimeStep \times (rooml/(Da \times Cvw) - Q \times (Xr\text{-}Xs)) / roomv$
$RH = Pa \times Xr \times Pg(Tr) \times 100 / (0.622 + Xr)$

Appendix C

1. PHYSICAL CONSTRAINTS OF CONTROL DEVICES

	Maximum value	Minimum value	Maximum change per sec
Supply fan	3000	10	100
Supply damper	4	3.093×10^{-3}	0.14
Chilled water flow	2.5×10^{-3}	8×10^{-6}	8.4×10^{-5}
Reheater	10	0	0.3
Humidifier	1	0	0.03

2. PID CONTROL (well tuned)

	Kp	Ki	Kd	Control aim
Supply fan	500	2	50	Keep P8 to P8s
Supply damper	0.18	3×10^{-3}	6	Keep Tr to Trs
Chilled water flow	1×10^{-3}	2×10^{-5}	5×10^{-4}	Keep Ts to Tss
Reheater	110	3×10^{-5}	500	Keep RH to RHs
Humidifier	1.3	3×10^{-3}	5	Keep RH to RHs

3. PID CONTROL (slightly detuned from optimal setting)

	Kp	Ki	Kd	Control aim
Supply fan	300	3	75	Keep P8 to P8s
Supply damper	0.11	5×10^{-3}	10	Keep Tr to Trs
Chilled water flow	0.5×10^{-3}	3×10^{-5}	7.5×10^{-4}	Keep Ts to Tss
Reheater	100	5×10^{-4}	750	Keep RH to RHs
Humidifier	1	5×10^{-3}	7.5	Keep RH to RHs

Appendix D

SYSTEM IDENTIFICATION CONTROL

	Model equations	Forgetting factor	Control aim
Supply fan	P8(t) = a*P8(t-1) + b*Ns(t)	0.95	Keep P8 to P8s
Supply damper	Tr(t) = a*Tr(t-1) + b*Tr(t-2) + c*Tr(t-3) + d*Tr(t-4) + e*Tr(t-5) + f*Rs(t)	0.95	Keep Tr to Trs
Chilled water flow	Ts(t) = a*Ts(t-1) + b*Ts(t-2) + c*Ts(t-3) + d*Ts(t-4) + e*Ts(t-5) + f*Qw(t)	0.95	Keep Ts to Tss
Reheater	P(t) = a*P(t-1) + b*P(t-2) + c*P(t-3) + d*RH(t)	0.95	Keep RH to RHs
Humidifier	WetF(t) = a*WetF(t-1) + b*WetF(t-2) + c*WetF(t-3) + d*RH(t)	0.95	Keep RH to RHs

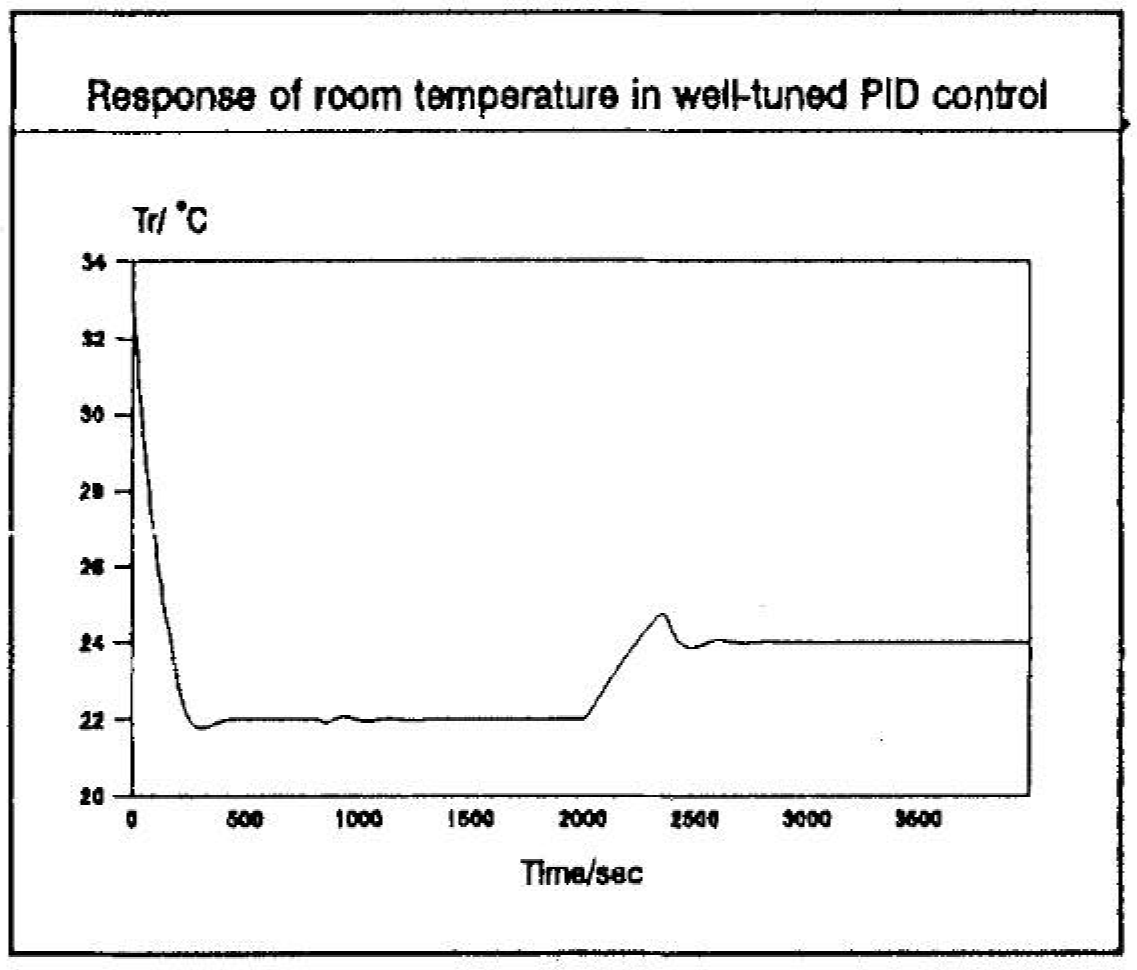

Figure 3(a)

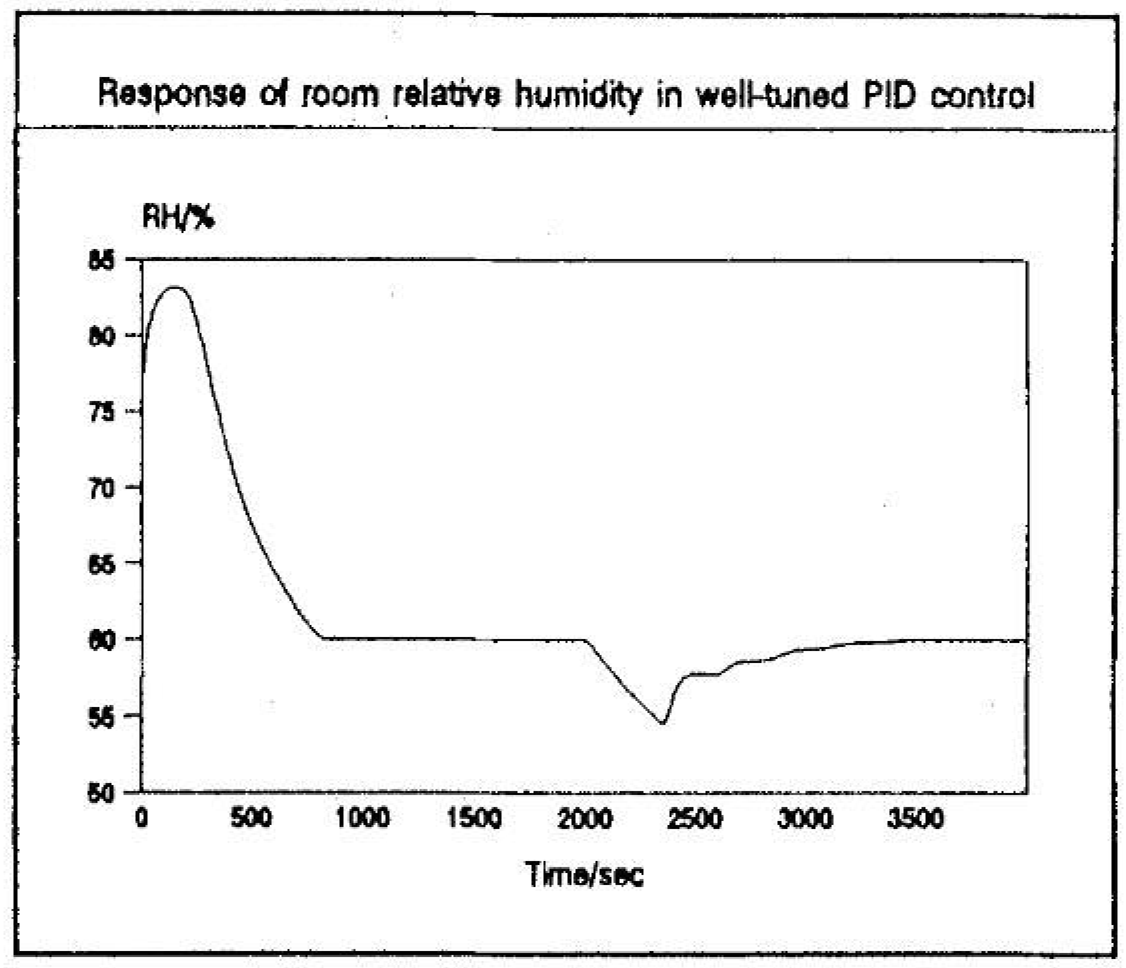

Figure 3(b)

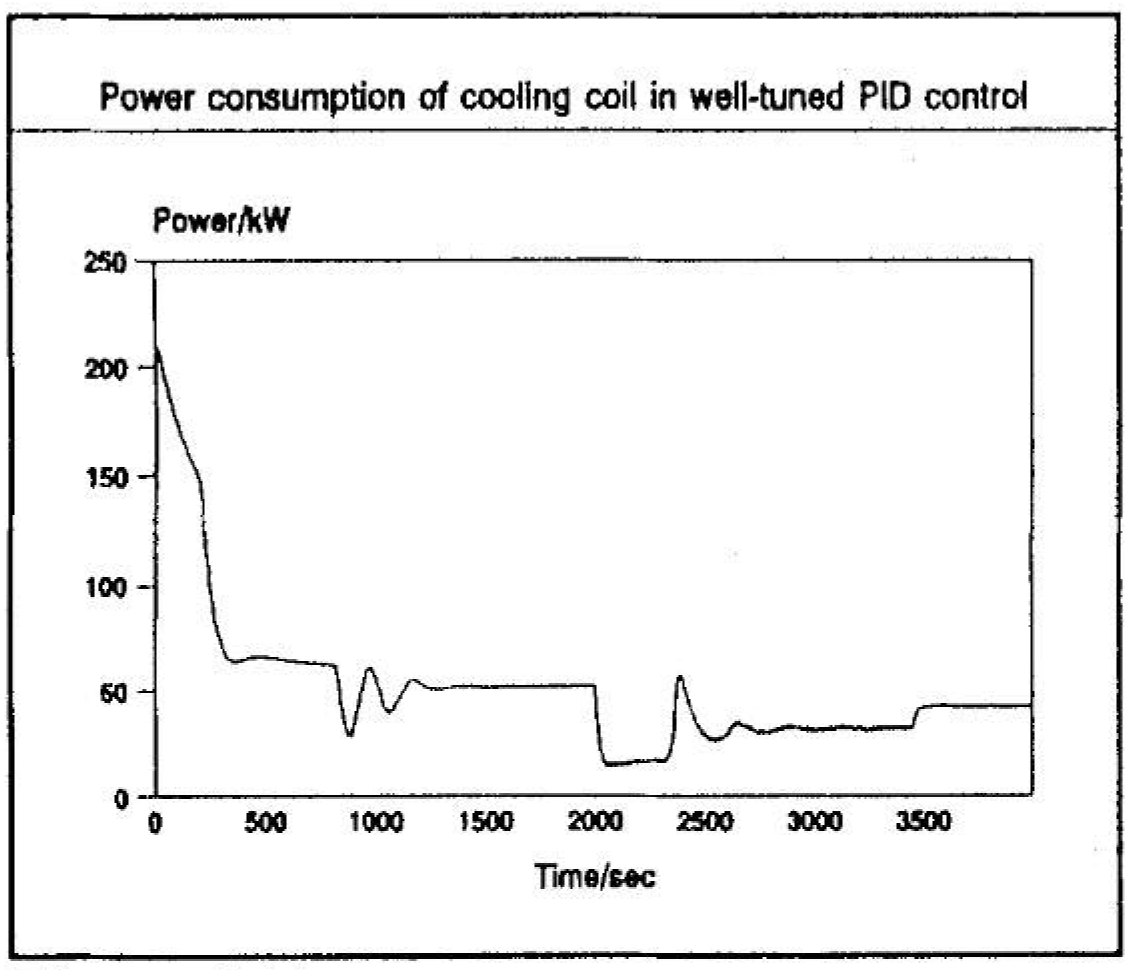

Figure 3(c)

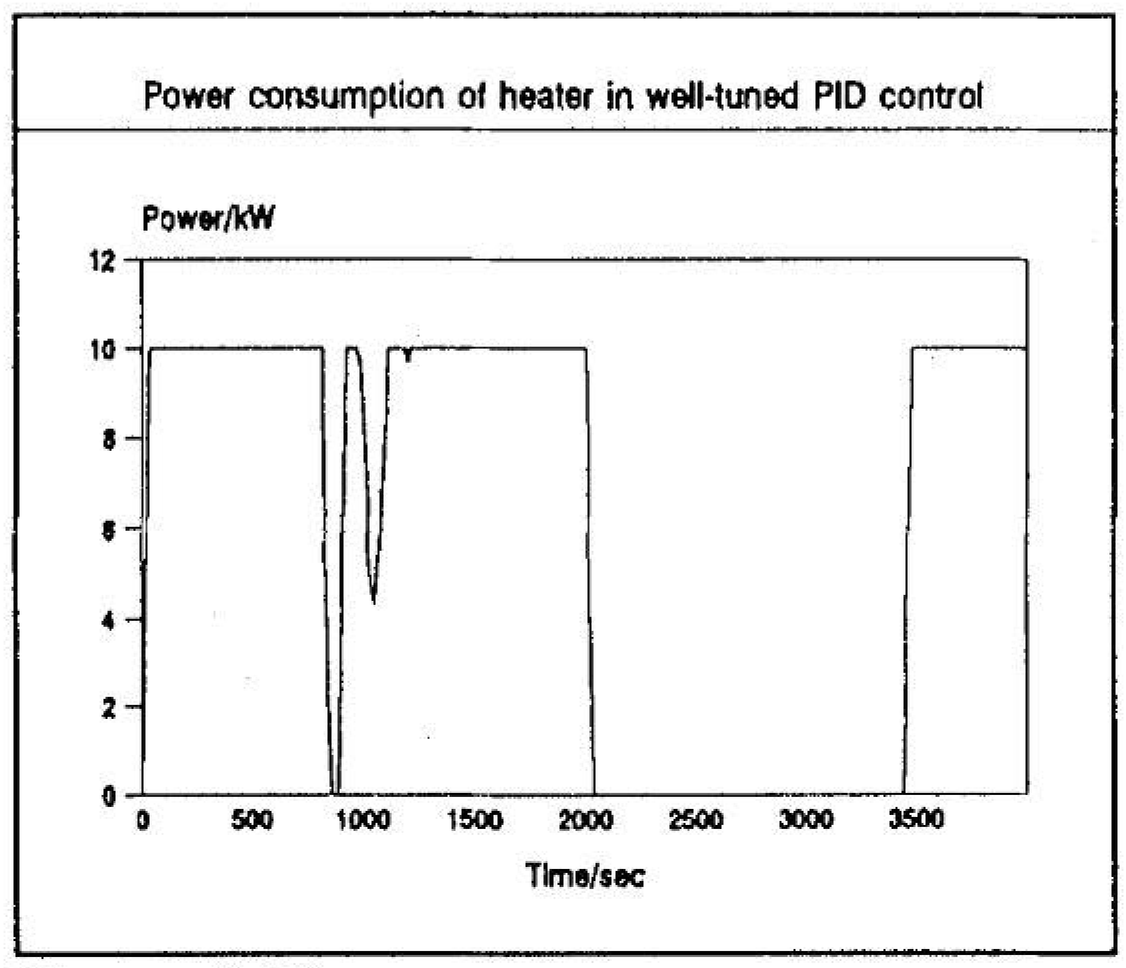

Figure 3(d)

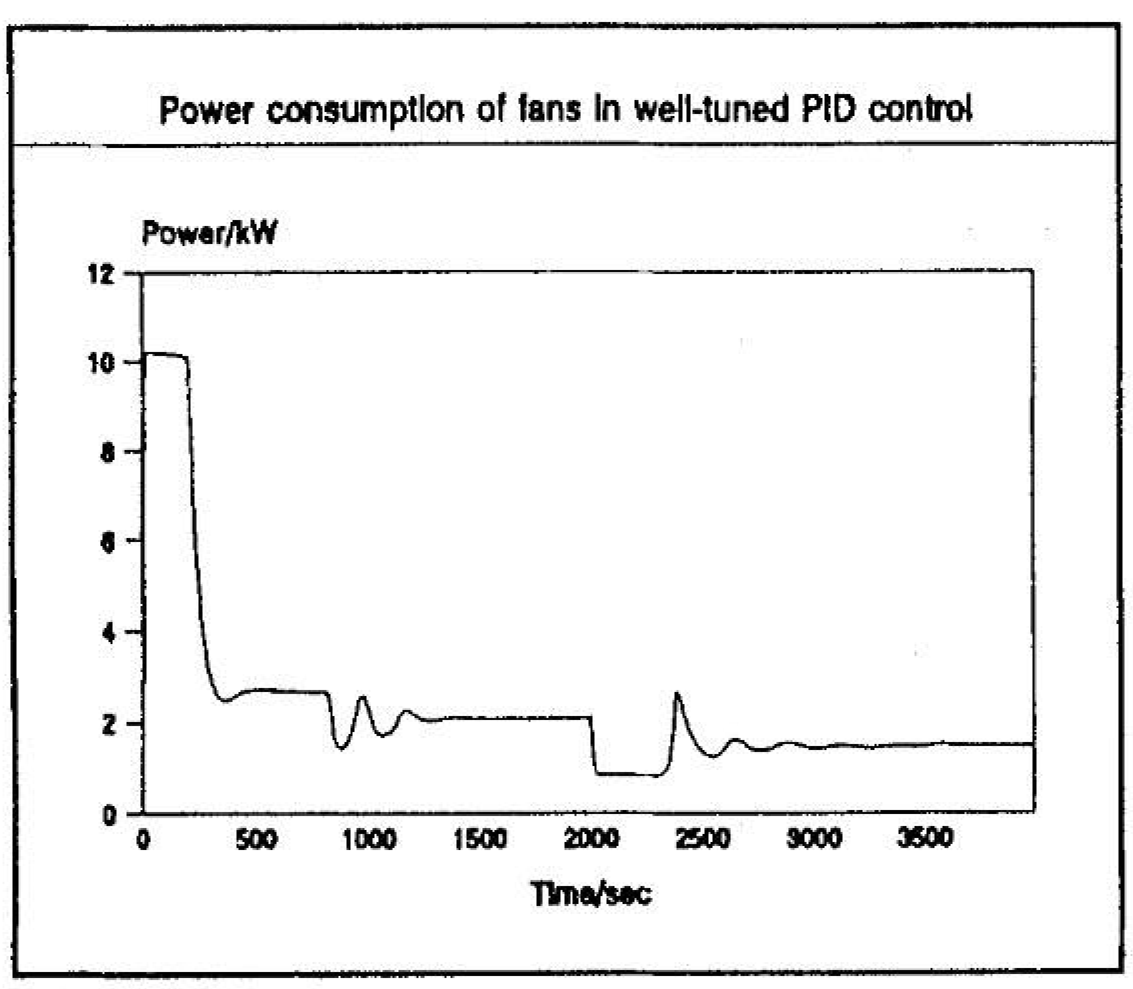

Figure 3(e)

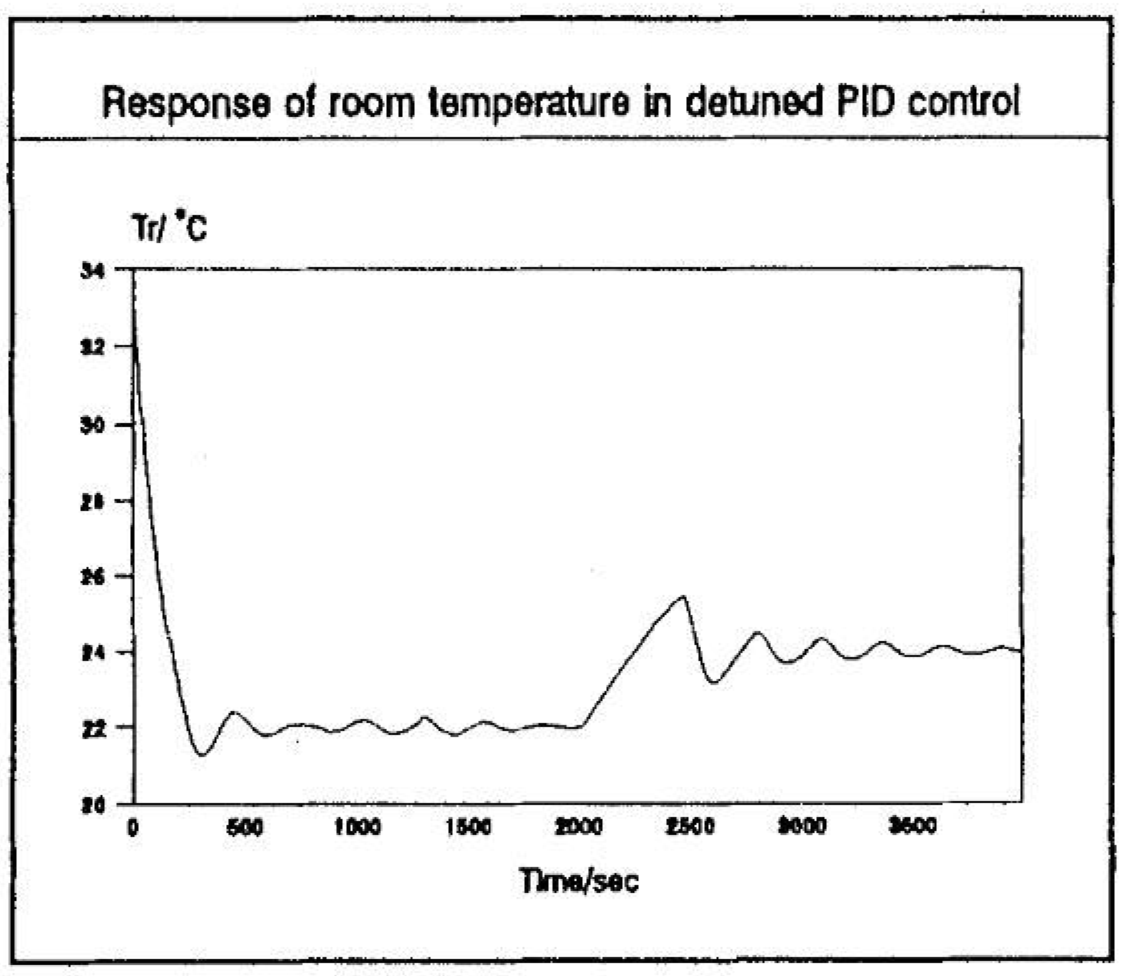

Figure 4(a)

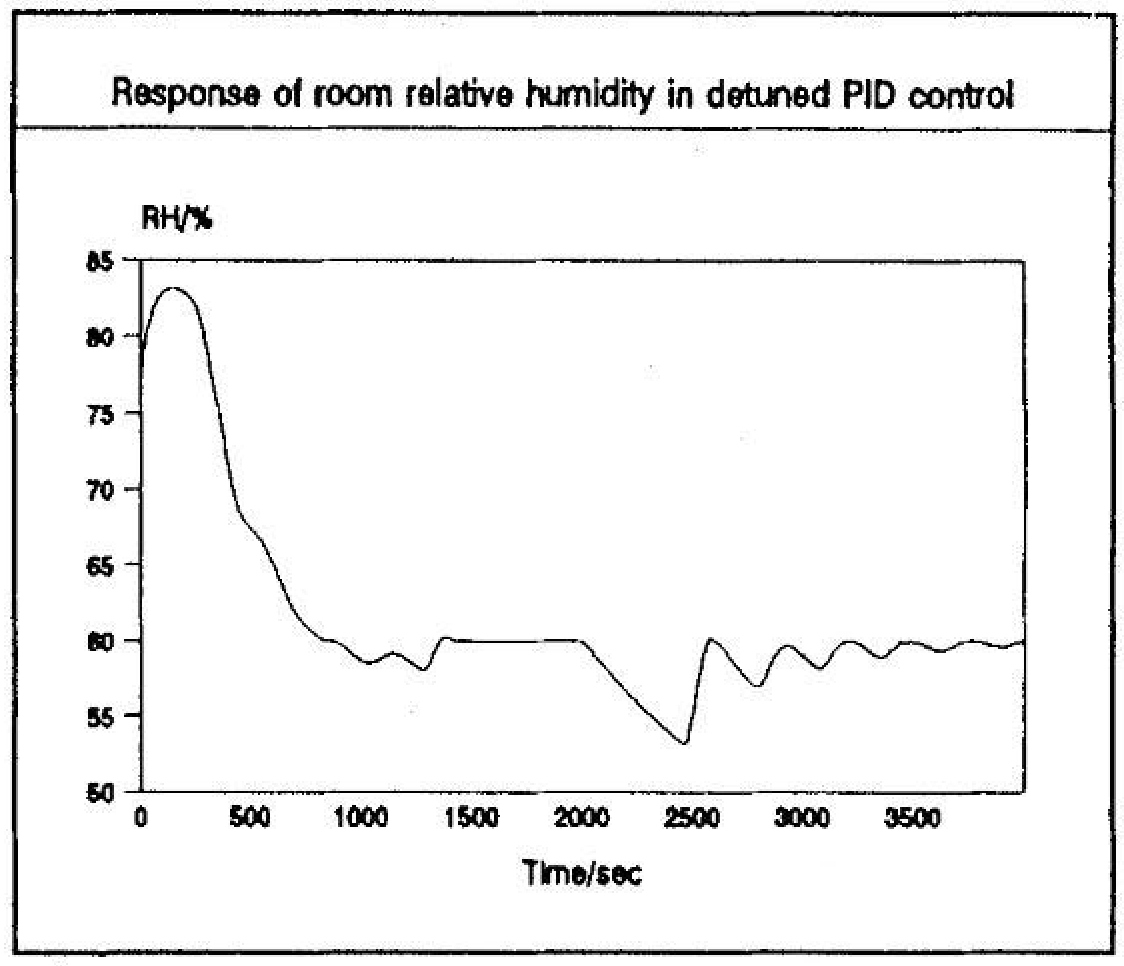

Figure 4(b)

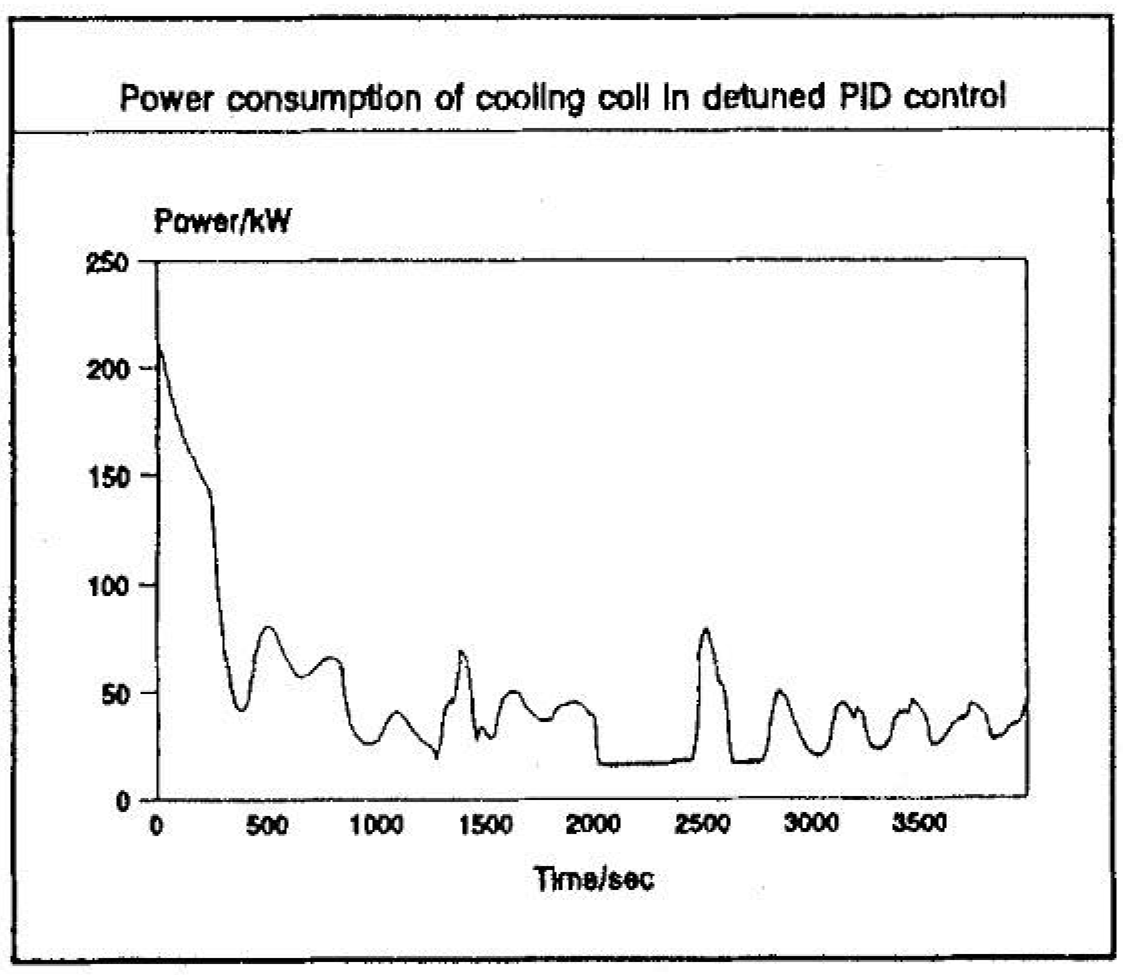

Figure 4(c)

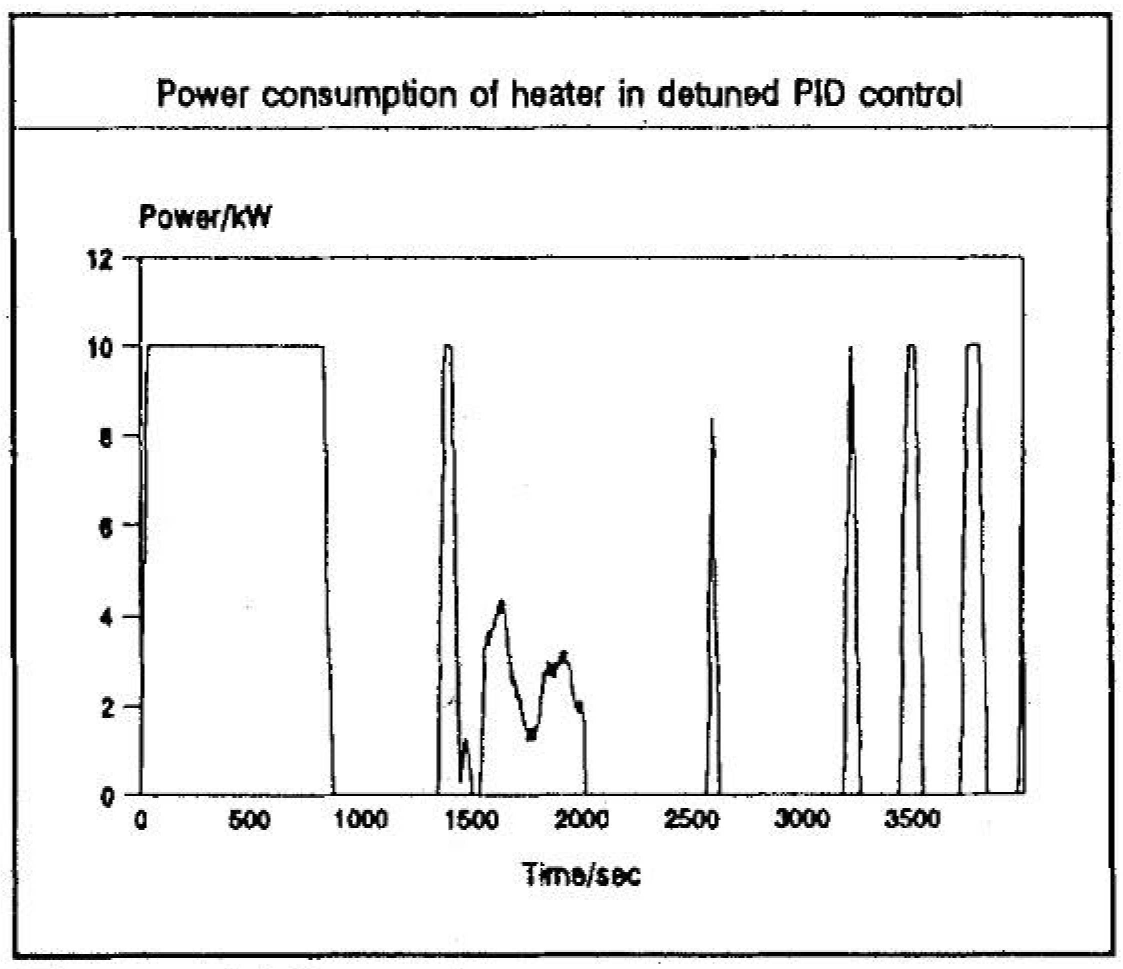

Figure 4(d)

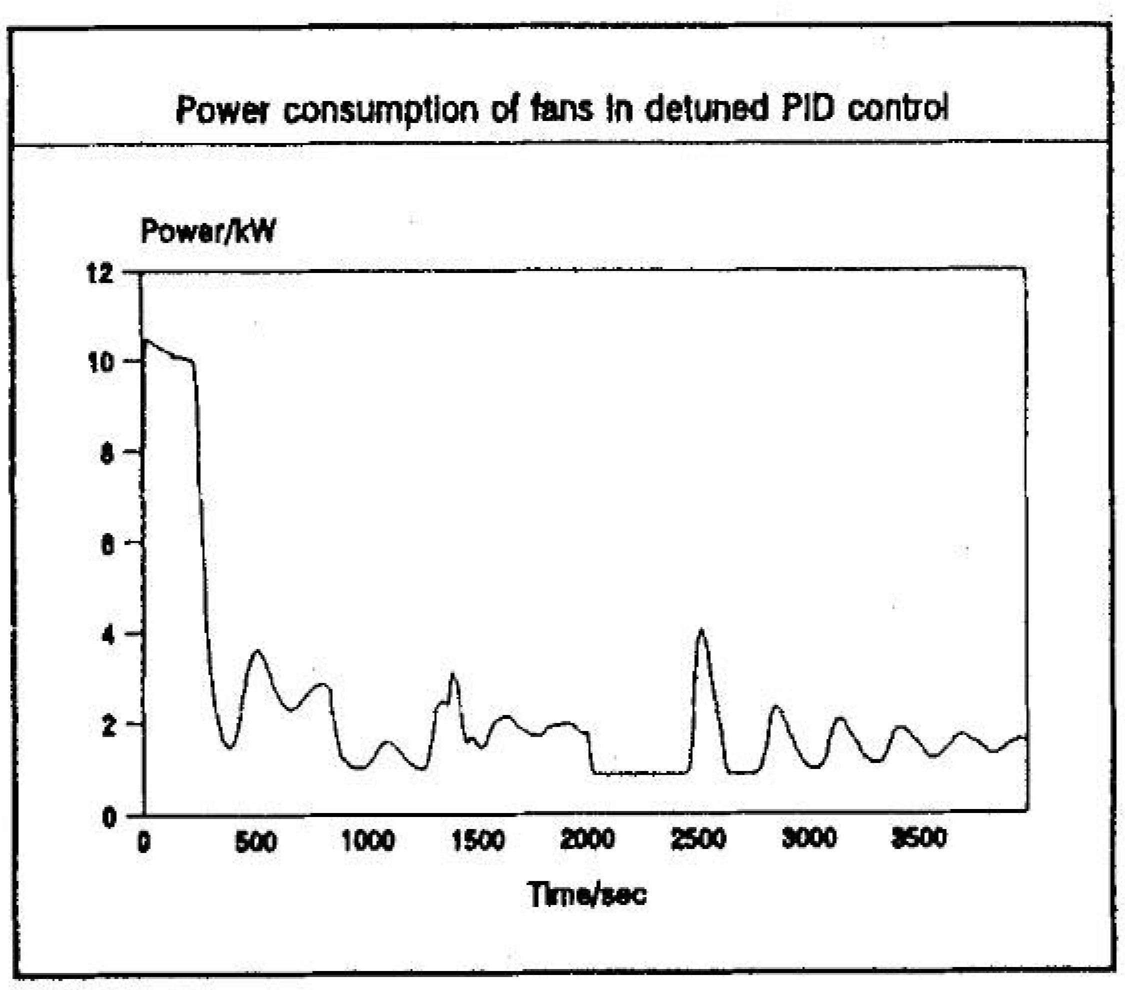

Figure 4(e)

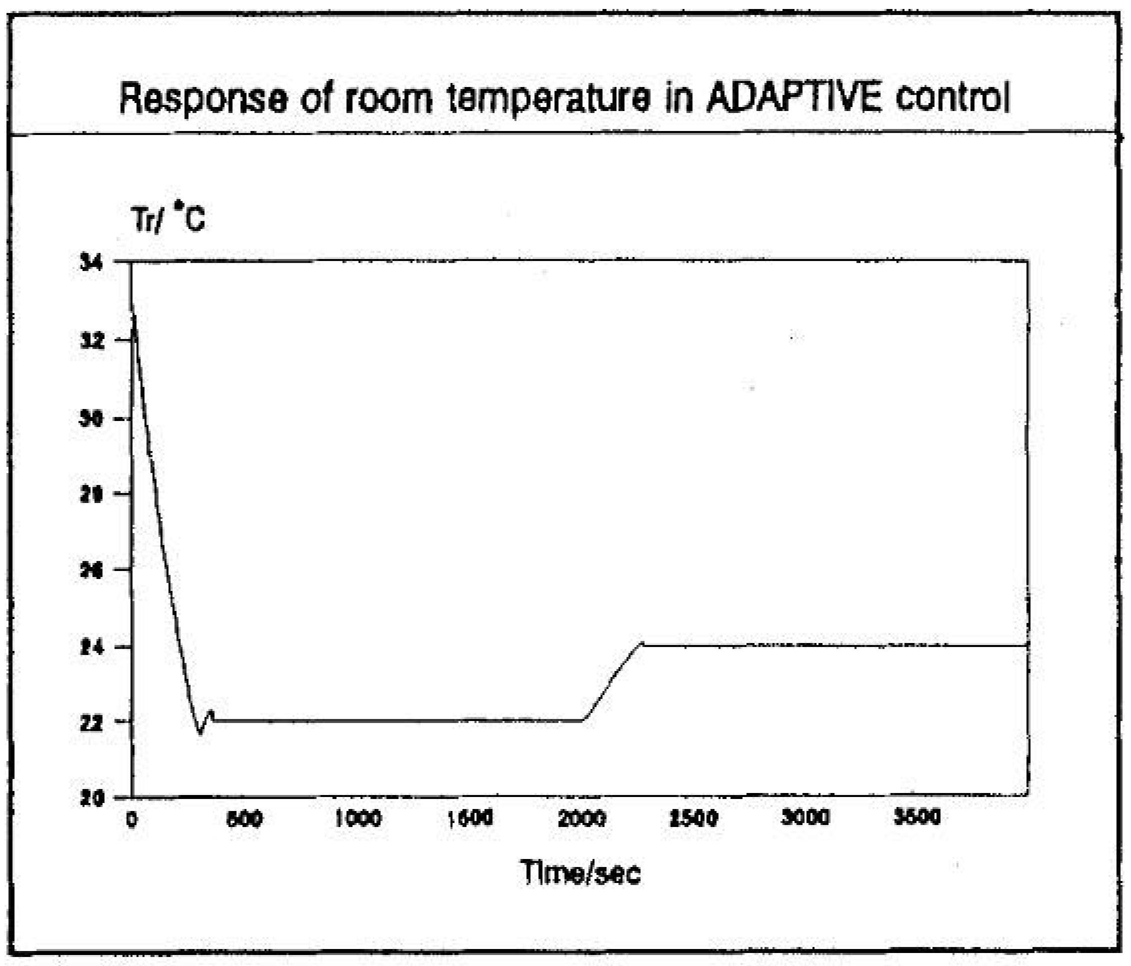

Figure 5(a)

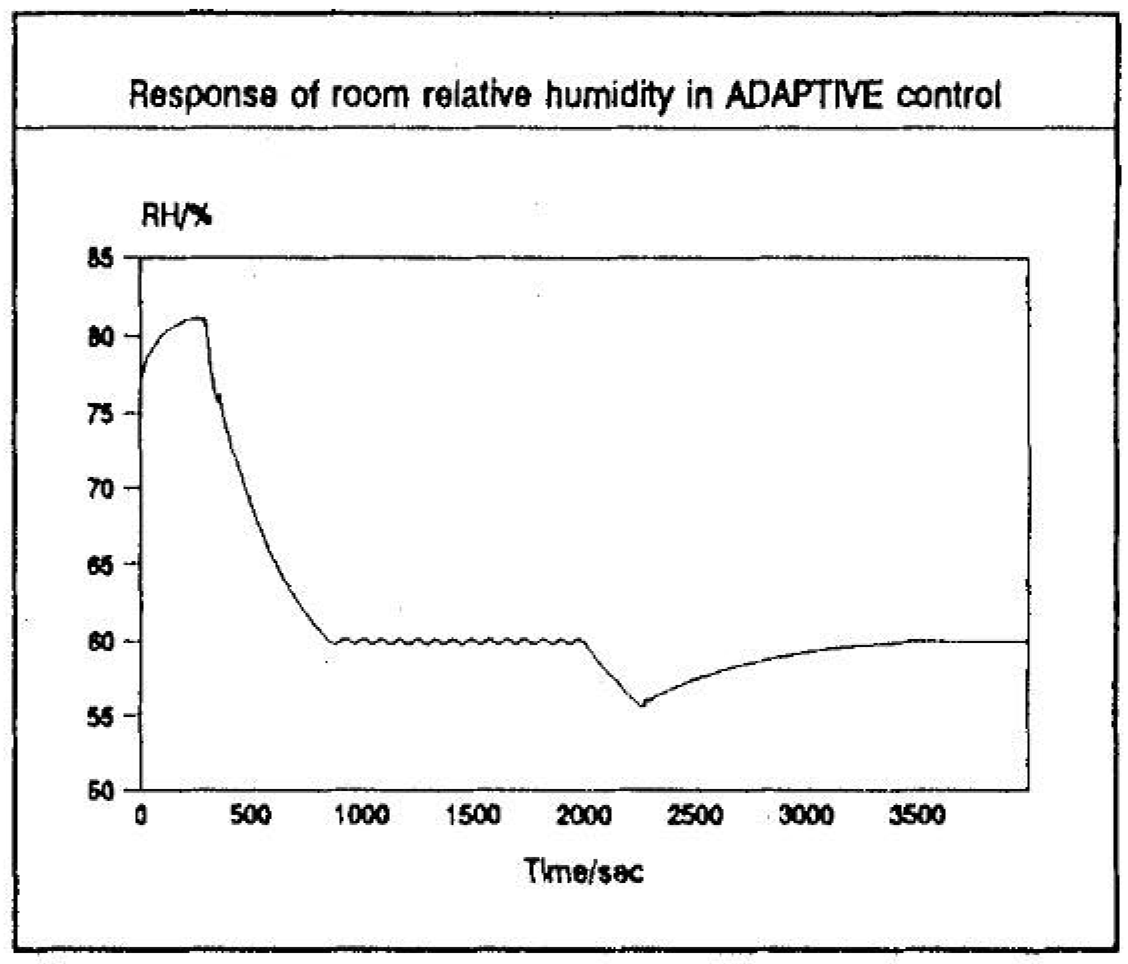

Figure 5(b)

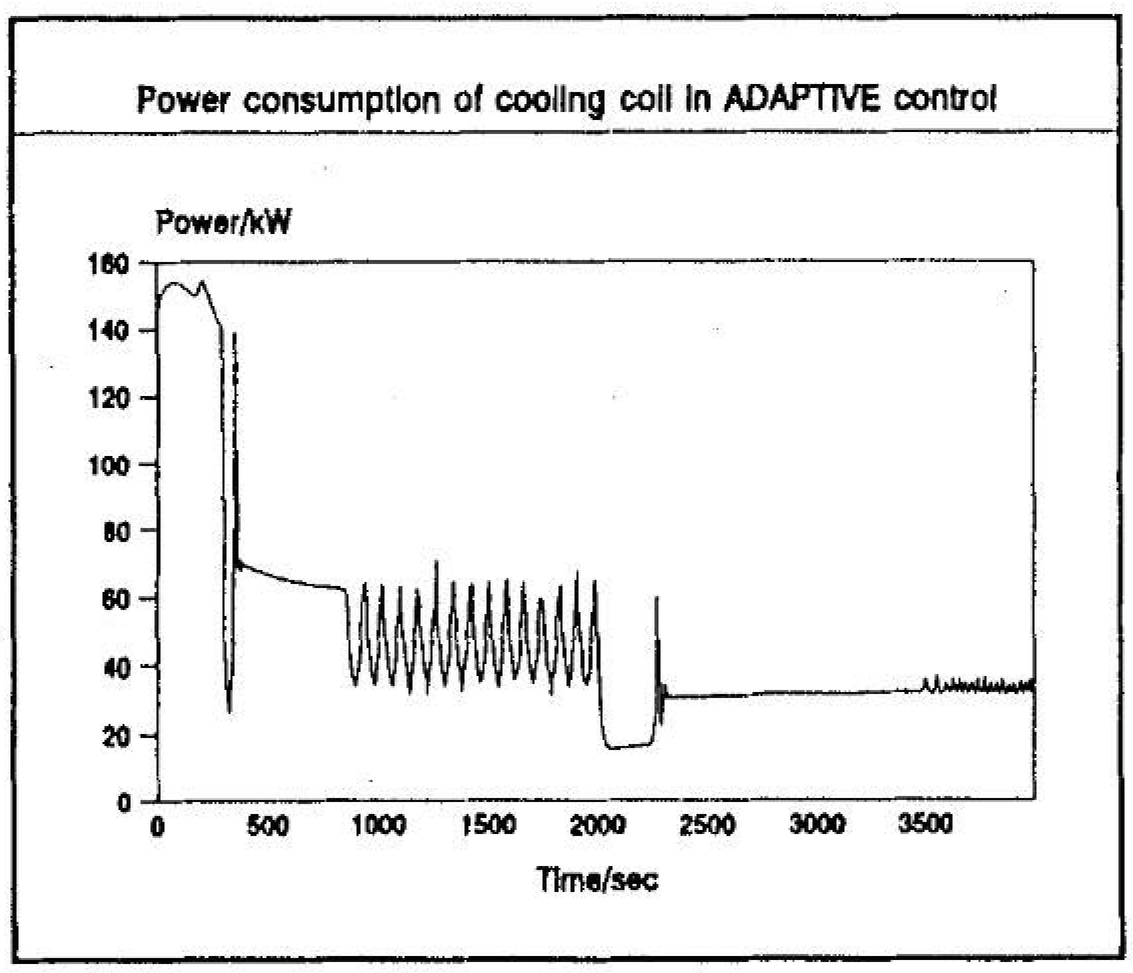

Figure 5(c)

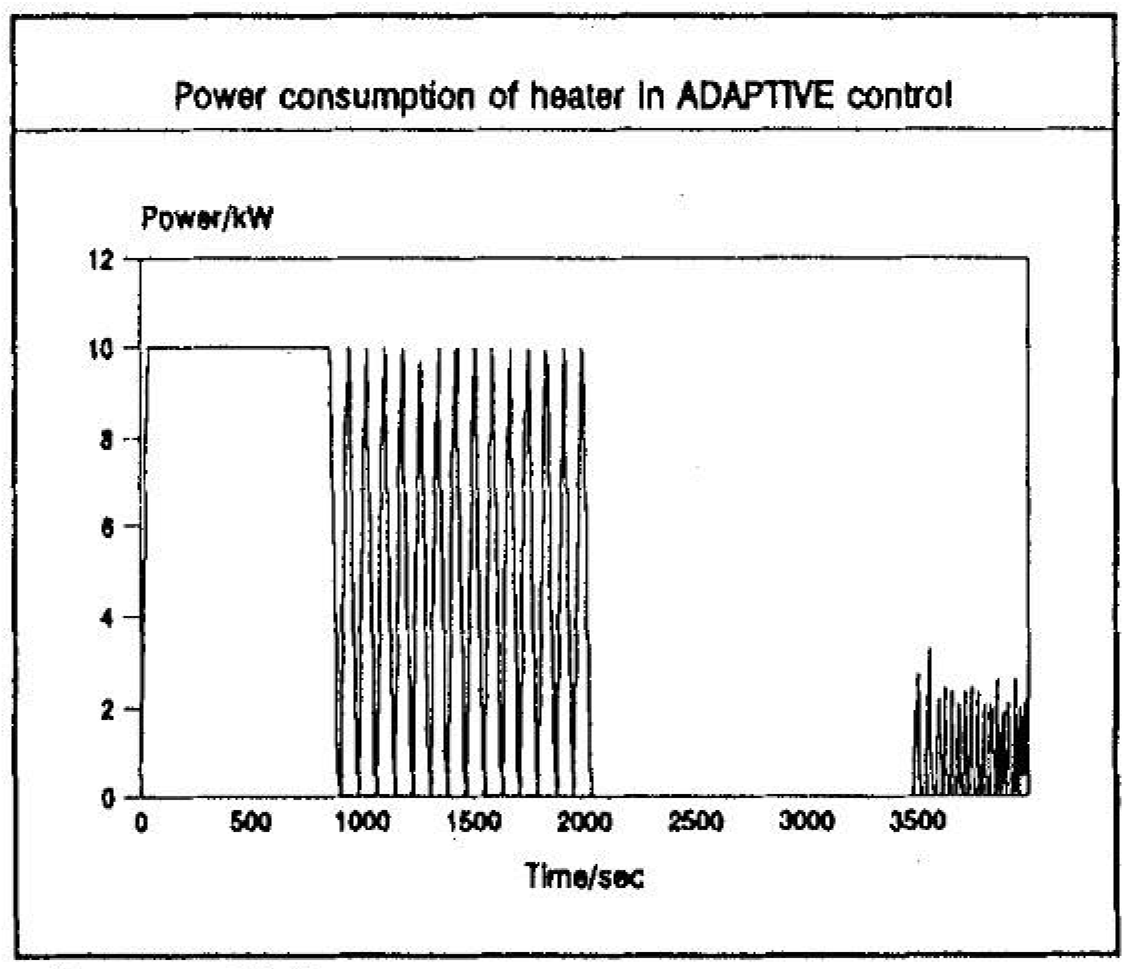

Figure 5(d)

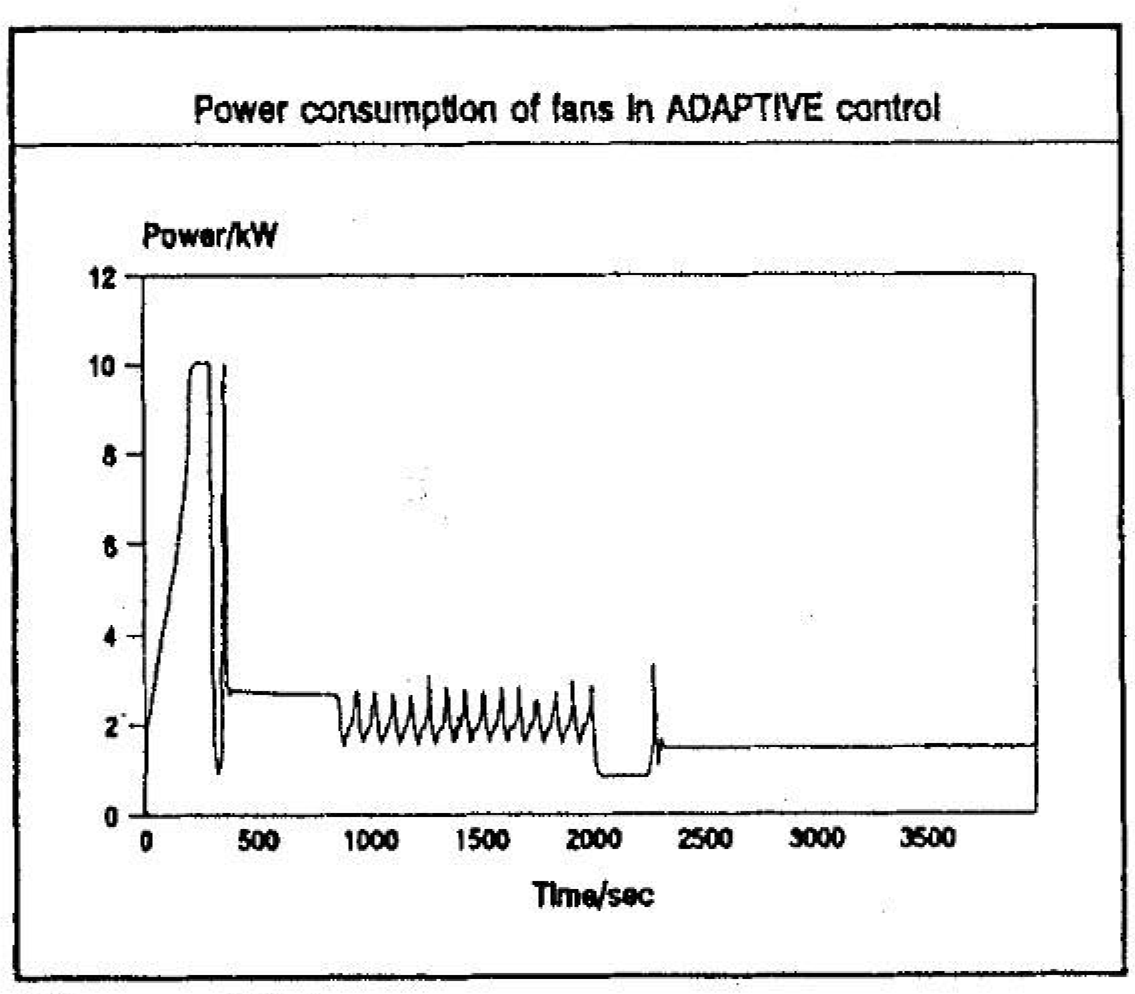

Figure 5(e)