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Valuation of distributed predictive information in robust economic dispatch

Rui Xie, Tao Tan and Yue Chen
Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Hong Kong SAR, People's Republic of China

ABSTRACT

Robust economic dispatch is an essential way to deal with the uncertainties of renewable generations, but its performance highly depends on the pre-built uncertainty set. This paper proposes a novel robust economic dispatch model in which the operator can buy predictions from distributed forecasters to build a better uncertainty set and enhance its dispatch efficiency. The value of distributed predictive information can be quantified by the operator's payment for buying it. The proposed model renders a two-stage robust optimisation problem with decision-dependent uncertainty (DDU). By analysing the structure of the uncertainty set, the proposed model is equivalently transformed into a two-stage robust optimisation problem with decision-independent uncertainty (DIU) so that it can be effectively solved by applying the column-and-constraint generation (C&CG) algorithm. Case studies show that the proposed method is effective.

KEYWORDS Robust optimisation; economic dispatch; renewable generation; distributed prediction; decision-dependent uncertainty
CONTACT Yue Chen yuechen@mae.cuhk.edu.hk
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1. Introduction

With the goal of mitigating global climate change, distributed renewable generators (DRGs) have witnessed rapid growth worldwide. This trend brings new challenges to the power system operation: On the one hand, the intermittent and fluctuating nature of DRGs such as photovoltaic panels and wind turbines leads to more uncertain power outputs and a higher risk of power imbalance. On the other hand, the power system operator may not have enough data to predict the DRG power outputs accurately due to the distributed feature of DRGs in terms of geography and ownership.

Robust economic dispatch is a typical way to hedge against uncertainty. Conventionally, the operator optimises the dispatch strategy against the worst-case scenario in a pre-built uncertainty set based on its own forecast. To list a few, a scenario-based robust economic dispatch method was proposed for active distribution networks with renewable integration (Zhang et al., 2019). To accelerate the computation of robust dynamic economic dispatch, a solution approach based on feasible region projection was proposed (Liu et al., 2020). The robust economic dispatch problem of integrated energy systems was studied and solved via sampling-based approximation (Lu et al., 2021). A scenario-based robust economic dispatch strategy was proposed for virtual power plants (Fang et al., 2021). For integrated transmission and distribution systems with renewables, a distributed robust economic dispatch method was developed based on the alternating direction method of multipliers (Chen et al., 2021).

However, since the choice of uncertainty set has a crucial impact on the robustness and optimality of the dispatch strategy (Ben-Tal and Nemirovski, 2002), using only the operator's forecast but ignoring the predictive information from other distributed forecasters may prevent the economic dispatch from achieving better performance. In this context, information markets (Chen et al., 2020) have been introduced to promote the use of distributed predictions.

In this paper, a novel robust economic dispatch model is proposed, in which the operator can decide on whether to buy the predictive information from distributed forecasters or not. The accuracy of distributed predictions can be evaluated via historical data and will influence how much the operator trusts the distributed predictions. After paying for the distributed predictive information, the operator combines the distributed predictions with its own forecasts to obtain an augmented uncertainty estimation with a higher accuracy and construct a more precise uncertainty set. The decision on whether to buy the distributed predictive information affects the range of the uncertainty set. In this way, decision-dependent uncertainty (DDU) arises in the proposed economic dispatch model considering information purchase decisions. The direct application of traditional solution algorithms such as Benders decomposition (Rahmaniani et al., 2017) and the column-and-constraint generation (C&CG) algorithm (Zeng and Zhao, 2013) fails to guarantee optimality. An effective solution method is needed.

The contributions of this paper are two-fold:

First, a robust economic dispatch model is developed in which the operator can use the predictive information from distributed forecasters after paying for them. An augmented uncertainty estimation, which is a weighted sum of the operator's own prediction and the distributed prediction, can be obtained and used to build the uncertainty set. It is shown that the purchased distributed predictions make the augmented uncertainty estimation more accurate and help to enhance the dispatch efficiency, but in the meantime, the operator needs to pay for the predictions. The proposed robust economic dispatch model characterises this trade-off and is formulated as a two-stage robust optimisation with DDU.

Second, a transformation-based method to solve the proposed robust economic dispatch model with DDU is developed. First, the uncertainty set is analysed and decomposed so that the proposed robust optimisation with DDU is transformed equivalently into a robust optimisation with decision-independent uncertainty (DIU). Thus, the transformed problem is of the form that the traditional C&CG algorithm can effectively handle, which means that the algorithm is guaranteed to converge and find the optimal solution. In the case studies, the proposed solution method is compared with the direct use of traditional C&CG to demonstrate its effectiveness and necessity.

The proposed approach differs from the authors' other work (Xie et al., 2024) in terms of modelling and solution methods. On the one hand, instead of assuming a payment-precision function for prediction and deciding the required precision (Xie et al., 2024), the proposed approach decides whether to buy the information or not, introducing integer variables which generally bring greater complexity than continuous ones to the optimisation problem. In this way, the payments at the purchase decision changing points can be used to quantify the value of the predictive information, as will be illustrated in the case study. On the other hand, rather than proposing a specialised algorithm to solve the two-stage robust optimisation with DDU (Xie et al., 2024), the proposed approach transforms the DDU into DIU, so that the traditional C&CG algorithm becomes effective for the equivalent problem. This solution approach is easier to understand and implement.

In the rest of this paper, we introduce the proposed robust economic dispatch model in Section 2 and develop the solution method in Section 3, which is verified by case studies in Section 4. The conclusion is summarised in Section 5.

2. Robust economic dispatch model

Suppose that there are controllable generators $j \in \mathcal{J} = \{1,2,\cdots,J\}$, DRGs $i \in \mathcal{I} = \{1,2,\cdots,I\}$, loads $s \in \mathcal{S} = \{1,2,\cdots,S\}$, and some distributed forecasters. The outputs of DRGs are uncertain and the operator can forecast the outputs using its own data or buying predictive information from distributed forecasters. For simplicity, we assume that there are I distributed forecasters, and each predicts the output of one DRG. In the following, we first build an uncertainty set for the outputs of DRGs considering the potential use of distributed predictive information. Then, a robust economic dispatch model integrating the purchase and use of distributed predictions is proposed.

2.1. Decision-dependent uncertainty set

Suppose that U_{it} is the random variable representing the output of DRG i in period $t \in \mathcal{T} = \{1,2,\cdots,T\}$. The system operator can calculate the expectation $\bar{u}_{it}$ and variance $\sigma^2_{U,it}$ of U_{it} based on historical data and use $\bar{u}_{it}$ as the DRG output prediction. At the same time, the distributed forecaster i can offer a forecast U^{pre}_{it} for U_{it} to the system operator. U^{pre}_{it} is also a random variable satisfying $U_{it} = U^{pre}_{it} + \varepsilon_{it}$, where ε_{it} denotes the random noise and its variance $\sigma^2_{\varepsilon,it}$ reflects the prediction accuracy. We assume that $\{\varepsilon_{it}; i \in \mathcal{I}, t \in \mathcal{T}\}$ are independent; the expectation $\mathbb{E}[\varepsilon_{it}] = 0$ and ε_{it} is independent of U_{it}.

If the operator buys the distributed predictive information from forecaster i, then an augmented forecast of U_{it} can be calculated using the best linear predictor, which is a weighted sum of the operator's own forecast $\bar{u}_{it}$ and the prediction U^{pre}_{it} bought from forecaster i, i.e.,

$$U^e_{it} = \frac{\sigma^2_{\varepsilon,it}}{\sigma^2_{U,it}+\sigma^2_{\varepsilon,it}}\bar{u}_{it} + \frac{\sigma^2_{U,it}}{\sigma^2_{U,it}+\sigma^2_{\varepsilon,it}}U^{pre}_{it} \,. \tag{1}$$

The weight is designed based on the variance of uncertain DRG output $\sigma^2_{U,it}$ and the variance of the prediction error of the distributed forecast $\sigma^2_{\varepsilon,it}$. The latter can be evaluated by comparing the distributed forecaster's historical predictions and the actual uncertainty realisations in hindsight. The weight shows how much the operator trusts the distributed predictions. The more trustworthy that the operator thinks the distributed prediction is (the smaller the $\sigma^2_{\varepsilon,it}$), the greater the weight on U^{pre}_{it}.

It is easy to prove that the covariance $\mathrm{cov}(U_{it} - U^e_{it}, U^{pre}_{it}) = 0$ (Xie et al., 2024). Based on this, we further assume that $U_{it} - U^e_{it}$ and U^{pre}_{it} are independent. Then, the variance of the prediction error of the augmented forecast U^e_{it} can be calculated as:

$$\mathrm{var}(U_{it} - U^e_{it} | U^{pre}_{it}) = \frac{\sigma^2_{\varepsilon,it}\sigma^2_{U,it}}{\sigma^2_{U,it}+\sigma^2_{\varepsilon,it}} \le \sigma^2_{U,it} \,. \tag{2}$$

The inequality indicates that after using the distributed predictive information, the variance of the prediction error decreases, and the prediction accuracy is enhanced.

Then, a binary variable z_i is introduced to indicate whether the operator buys the prediction from forecaster i. Particularly, $z_i = 1$ if the operator buys the prediction from forecaster i and $z_i = 0$ otherwise. Denote the realisation of U^{pre}_{it} by u^{pre}_{it} and let:

$$u^0_{it} := z_i\left(\frac{\sigma^2_{\varepsilon,it}}{\sigma^2_{U,it}+\sigma^2_{\varepsilon,it}}\bar{u}_{it} + \frac{\sigma^2_{U,it}}{\sigma^2_{U,it}+\sigma^2_{\varepsilon,it}}u^{pre}_{it}\right) + (1 - z_i)\bar{u}_{it} \,, \tag{3}$$

$$u^h_{it} := z_i\sqrt{\mathrm{var}(U_{it} - U^e_{it}|U^{pre}_{it})/\delta} + (1 - z_i)\sqrt{\sigma^2_{U,it}/\delta} \,. \tag{4}$$

Suppose that the errors $\{(U_{it} - U^e_{it})/\sqrt{\mathrm{var}(U_{it} - U^e_{it})}; i \in \mathcal{I}, t \in \mathcal{T}\}$ are independent and identically distributed (i.i.d.). Using Chebyshev inequality (Grimmett and Stirzaker, 2020), we can prove that for any $\delta, \xi \in (0,1)$,

$$\mathcal{P}(|U_{it} - u_{it}^0| \geq u_{it}^h) \leq \delta\ , \tag{5}$$

$$\mathcal{P}(\sum_{i,t}|U_{it} - u_{it}^0|/u_{it}^h \geq \Gamma) \leq \xi\ , \tag{6}$$

where $\mathcal{P}(\cdot)$ is the probability measure and

$$\Gamma := \sqrt{IT\delta(1+IT\xi)/\xi}\ , \tag{7}$$

is the uncertainty budget (Bertsimas and Sim, 2004).

Then, we can build a polyhedral uncertainty set:

$$\begin{aligned}\mathcal{U}(z) := \{u_{it}; i \in \mathcal{I}, t \in \mathcal{T}: |u_{it} - u_{it}^0| \leq u_{it}^h,\\ \textstyle\sum_{i,t}|u_{it} - u_{it}^0|/u_{it}^h \leq \Gamma\}.\end{aligned} \tag{8}$$

When δ and ξ are small enough, the uncertain output will be in $\mathcal{U}(z)$ with a high probability. We use the notation $\mathcal{U}(z)$ to emphasise that it depends on the information purchase decision.

2.2. Two-stage robust economic dispatch

With the above uncertainty set, a two-stage robust economic dispatch problem integrating the purchase and use of distributed predictive information can be developed. In the day-ahead pre-dispatch stage, the operator decides generator outputs, reserves, and whether to buy the predictive information from distributed forecasters. The uncertain output of DRGs is realised after the pre-dispatch stage and before the re-dispatch stage. In the re-dispatch stage, the operator adjusts the generators for power balance. The two-stage robust economic dispatch problem can be formulated as:

$$\min_{z,p,r} \sum_i C_i z_i + f(p,r) + \max_{u \in \mathcal{U}(z)} \min_{y \in \mathcal{Y}(p,r,u)} g(y) \tag{9a}$$

$$\text{s.t.}\ \ (p,r) \in \mathcal{X}, z_i \in \{0,1\}, \forall i \in \mathcal{I}\ , \tag{9b}$$

where C_i is the payment for buying the prediction from distributed forecaster i; the pre-dispatch variables include the output p_{jt} and reserve r_{jt} of generator j in period t, for all $j \in \mathcal{J}$ and $t \in \mathcal{T}$; $\mathcal{X}$ is the feasible region of the pre-dispatch variables; $f(p,r)$ is the operational cost of the pre-dispatch stage; the re-dispatch variable is the output adjustment y; $\mathcal{Y}(p,r,u)$ is the feasible region of y, which depends on the pre-dispatch variables and the uncertainty realisations; $g(y)$ is the operational cost of the re-dispatch stage. Therefore, Equation (9a) represents the worst-case total cost of the two stages. The definitions of $f(p,r)$, $g(y)$, $\mathcal{X}$, and $\mathcal{Y}(p,r,u)$ are given in detail below:

$$f(p,r) := \sum_t \sum_j (\rho_j p_{jt} + \gamma_j r_{jt})\,\Delta_t\ , \tag{10}$$

$$g(y) := \sum_t \sum_j \rho_j y_{jt} \Delta_t\ , \tag{11}$$

$$\begin{aligned}\mathcal{X} := \{(p,r): &\textstyle\sum_j p_{jt} + \sum_i u_{it}^0 - \sum_s p_{st}^d\\ &= \textstyle\sum_j L_j p_{jt} + \sum_i L_i u_{it}^0 - \sum_s L_s p_{st}^d + L_0\end{aligned} \tag{12a}$$

$$P_j^{\min} + r_{jt} \leq p_{jt} \leq P_j^{\max} - r_{jt}, 0 \leq r_{jt} \leq R_j^{\max} \tag{12b}$$

$$\begin{aligned}-R_j^{\max}\Delta_t \leq (p_{jt} + r_{jt}) - (p_{j(t-1)} - r_{j(t-1)})\\ \leq R_j^{\max}\Delta_t\end{aligned} \tag{12c}$$

$$\begin{aligned}-R_j^{\max}\Delta_t \leq (p_{jt} - r_{jt}) - (p_{j(t-1)} + r_{j(t-1)})\\ \leq R_j^{\max}\Delta_t\end{aligned} \tag{12d}$$

$$-F_l \leq \sum_j \pi_{jl} p_{jt} + \sum_i \pi_{il} u_{it}^0 - \sum_s \pi_{sl} p_{st}^d \leq F_l\}\ , \tag{12e}$$

$$\mathcal{Y}(p,r,u) := \{y: -r_{jt} \leq y_{jt} \leq r_{jt} \tag{13a}$$

$$\begin{aligned}&\textstyle\sum_j (p_{jt} + y_{jt}) + \sum_i u_{it} - \sum_s p_{st}^d\\ &= \textstyle\sum_j L_j (p_{jt} + y_{jt}) + \sum_i L_i u_{it} - \sum_s L_s p_{st}^d + L_0\end{aligned} \tag{13b}$$

$$-F_l \leq \sum_j \pi_{jl}(p_{jt} + y_{jt}) + \sum_i \pi_{il} u_{it} - \sum_s \pi_{sl} p_{st}^d \leq F_l\}\ . \tag{13c}$$

In Equation (10), ρ_j and γ_j are the unit costs of power generation and reserve capacity of generator j, respectively; Δ_t is the period length. Constraint (12a) is the power balance equation considering transmission losses, where L_j, L_i, and L_s are the loss sensitivity coefficients at the corresponding buses (Litvinov et al., 2004). For example, L_s is the loss sensitivity coefficient at the bus of the load demand p_{st}^d. L_0 is the system loss offset (Litvinov et al., 2004). Constraints (12b) - (12d) are for the operation of generators. Constraint (12b) contains the generation power limits and the bounds for the reserve capacity. The ramp rate bounds are in constraints (12c) and (12d). F_l is the line capacity. π_{jl}, π_{il}, and π_{sl} are power transfer distribution factors. Thus, constraint (12e) bounds the line power. Similarly, constraints (13a) - (13c) are for the power adjustment bounds, the power balance considering transmission losses, and the line flow bounds, respectively.

The proposed model (9) is difficult to solve because it is a two-stage robust optimisation problem with DDU, where the uncertainty set $\mathcal{U}(z)$ depends on the first-stage binary variable z. The traditional algorithms for two-stage robust optimisation with DIU are not effective for such a problem.

3. Solution method

In this section, an effective solution method is developed. First, we analyse the structure of the uncertainty set $\mathcal{U}(z)$ and decompose it so that model (9) can be transformed into a two-stage robust optimisation problem with DIU. Then, the C&CG algorithm is applied to solve the problem effectively.

3.1. DDU transformation

From the uncertainty set defined in Equation (8), we observe that:

$$\mathcal{U}(z) = \{u_{it} = u^0_{it}(z) + u^h_{it}(z) \cdot \phi_{it}; i \in \mathcal{I}, t \in \mathcal{T}: |\phi_{it}| \le 1, \forall i, \forall t, \textstyle\sum_{i,t}|\phi_{it}| \le \Gamma\}, \tag{14}$$

where ϕ_{it} is an auxiliary variable to represent $|u_{it} - u^0_{it}|/u^h_{it}$; $u^0_{it}(z)$ and $u^h_{it}(z)$ are defined in Equations (3) and (4). In this regard, we construct an uncertainty set for ϕ separately as:

$$\begin{aligned}\Phi := \{\phi_{it}; i \in \mathcal{I}, t \in \mathcal{T}: |\phi_{it}| \le 1, \textstyle\sum_{i,t}|\phi_{it}| \le \Gamma\} \\ = \{\phi_{it}; i \in \mathcal{I}, t \in \mathcal{T}: \phi_{it} \le \psi_{it}, \phi_{it} \le -\psi_{it},\end{aligned} \tag{15a}$$

$$\psi_{it} \le 1, \forall i, \forall t, \textstyle\sum_{i,t}\psi_{it} \le \Gamma\}, \tag{15b}$$

where ψ_{it} represents $|\phi_{it}|$ and Φ is a polyhedron according to Equation (15b). Then by Equation (14), $u \in \mathcal{U}(z)$ is equivalent to $u_{it} = u^0_{it}(z) + u^h_{it}(z) \cdot \phi_{it}$ for all $i \in \mathcal{I}, t \in \mathcal{T}$ and some $\phi \in \Phi$. In this way, the DDU is decomposed into the DIU ϕ and the decision-dependent coefficients $u^0_{it}(z)$ and $u^h_{it}(z)$. Then the proposed model (9) is equivalently transformed into:

$$\min_{z,p,r} \textstyle\sum_i C_i z_i + f(p,r) + \max_{\phi \in \Phi} \min_{y \in \mathcal{Y}'(z,p,r,\phi)} g(y), \tag{16a}$$

$$\text{s.t. } (p,r) \in \mathcal{X}, z_i \in \{0,1\}, \forall i \in \mathcal{I}, \tag{16b}$$

with the new second-stage feasible region

$$\mathcal{Y}'(z,p,r,\phi) := \{y: (13), u = u^0(z) + u^h(z) \cdot \phi\}. \tag{17}$$

3.2. C&CG solution algorithm

Problem (16) is a two-stage robust optimisation problem with DIU, so it can be effectively solved by the C&CG algorithm, and the algorithm converges to an optimal solution. For conciseness, we denote the pre-dispatch variables by $x := (z,p,r)$ and write $g(y)$ and $\mathcal{Y}'(x,\phi) = \mathcal{Y}'(z,p,r,\phi)$ in compact forms as:

$$g(y) = c^T y, \tag{18}$$

$$\mathcal{Y}'(x,\phi) = \{y: Ay \ge B(x) \cdot \phi + Dx + q\}, \tag{19}$$

where the coefficients A, c, D, and q are constant matrices or vectors and $B(x)$ is a matrix linear in x. Then, the C&CG algorithm (Zeng and Zhao, 2013) for solving problem (16) is as follows.

Algorithm 1: C&CG algorithm

Input: Parameters of problem (16); error tolerance $\varepsilon > 0$.

Output: Solution x^{K+1} and optimal value UB.

Step 1. Initiation: $K = 0$; $\text{UB} = +\infty$.

Step 2. Solve the primary problem as follows:

$$\min_{x=(z,p,r),\zeta,y^k} \textstyle\sum_i C_i z_i + f(p,r) + \zeta \tag{20a}$$

$$\text{s.t. } (p,r) \in \mathcal{X}, z_i \in \{0,1\}, \forall i \in \mathcal{I} \tag{20b}$$

$$\zeta \ge c^T y^k, k = 1, \cdots, K \tag{20c}$$

$$Ay^k \ge B(x) \cdot \phi^k + Dx + q, k = 1, \cdots, K. \tag{20d}$$

Let x^K be the optimal solution and LB be the optimal objective value.

Step 3. Solve the feasibility-check problem as follows:

$$\max_{\phi,y,s,\mu} 1^T s, \tag{21a}$$

$$\text{s.t. } \phi \in \Phi, A^T\mu = 0, \tag{21b}$$

$$0 \le \mu \perp (Ay + s - B(x^{K+1}) \cdot \phi - Dx - q) \ge 0, \tag{21c}$$

$$0 \le s \perp (1 - \mu) \ge 0.. \tag{21d}$$

Let ϕ^{K+1} be the optimal solution. If the optimal value exceeds 0, then let $K = K + 1$ and go to Step 2. Otherwise, go to Step 4.

Step 4. Solve the secondary problem as follows:

$$\max_{\phi,y,\mu} c^T y, \tag{22a}$$

$$\text{s.t. } \phi \in \Phi, A^T\mu = c, \tag{22b}$$

$$0 \le \mu \perp (Ay - B(x^{K+1}) \cdot \phi - Dx - q) \ge 0. \tag{22c}$$

Let ϕ^{K+1} be the optimal solution and let UB equal $\sum_i C_i z_i^{K+1} + f(p^{K+1}, r^{K+1})$ plus the optimal value.

Step 5. $K = K + 1$. If $|\text{UB} - \text{LB}| \le \varepsilon$, terminate and output x^K and UB. Otherwise, go to Step 2.

Since $B(x)$ is linear in x, problem (20) is a mixed-integer linear program (MILP). Problem (21) and problem (22) have complementarity constraints (21c), (21d), and (22c), which can be transformed into linear constraints using auxiliary binary variables and the Big-M technique (Pineda and Morales, 2019). For example, constraint (21d) is equivalent to:

$$\begin{cases} 0 \le s \le M(1 - v) \\ 0 \le 1 - \mu \le Mv \\ v \text{ is binary} \end{cases}, \tag{23}$$

with a large enough positive constant M. In this way, problems (21) and (22) can also be effectively solved by commercial MILP solvers.

4. Case study

In this section, we examine the effectiveness of the proposed method on a five-bus power system. Then, the value of distributed predictive information is investigated via sensitivity analysis of the payment for buying distributed predictions. The proposed solution method is also compared with the direct adoption of the C&CG algorithm to show its necessity. All MILP problems are solved by Gurobi 9.5.

4.1. Benchmark

There are two generators and three DRGs in the test power system, as drawn in Figure 1. The original forecasts (solid lines) and the actual values (dashed lines) of the DRG output are depicted in Figure 2. The variance in the DRG output is $\sigma_U^2 = (0.08, 0.02, 0.04)$ MW^2, and the variance in the errors of the corresponding distributed forecasts is set as $\sigma_\varepsilon^2 = \sigma_U^2/2$. Other parameters are listed in Table 1.

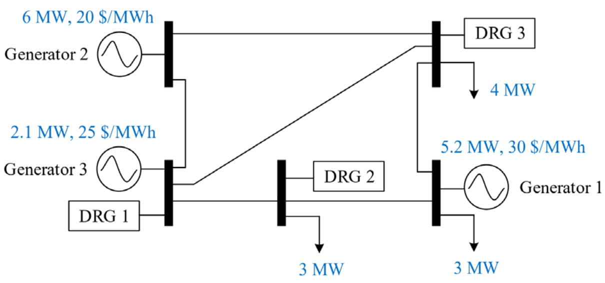

Figure 1. The five-bus test power system.

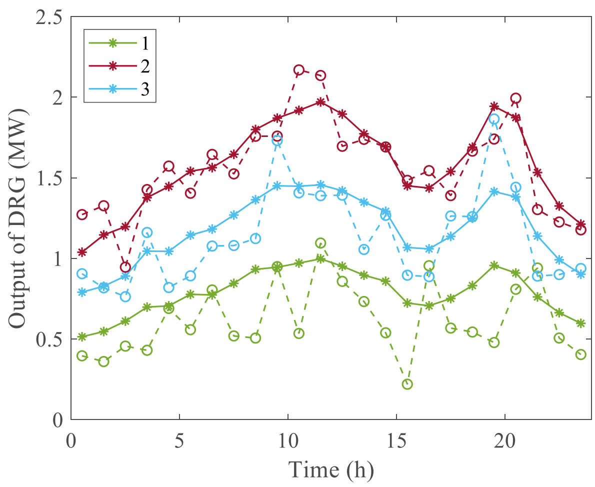

Figure 2. DRG output (solid lines for original forecasts and dashed lines for actual values).

Table 1. Parameters.

Parameter	Value	Parameter	Value
T	24	Δ_t	1 h
δ	0.05	ξ	0.05
C_i	US$40	ε	US$1

The proposed solution method is applied, and the optimal solution is obtained after 20 iterations. The total computational time is about 80 seconds, which is acceptable for economic dispatch purposes. As the result shows, the system operator decides to buy the distributed predictive information of DRG 1 and DRG 3, with a total payment of US$80. The operational cost is US$2,833, so the total cost is US$2,913. In contrast, if the operator cannot make use of the distributed predictive information (all $z_i, \forall i$ are forced to be zero), the operational cost is US$2,983. Therefore, the utilisation of distributed predictive information helps to reduce the operational cost. The proposed method is effective to find the optimal solution with the minimum total cost.

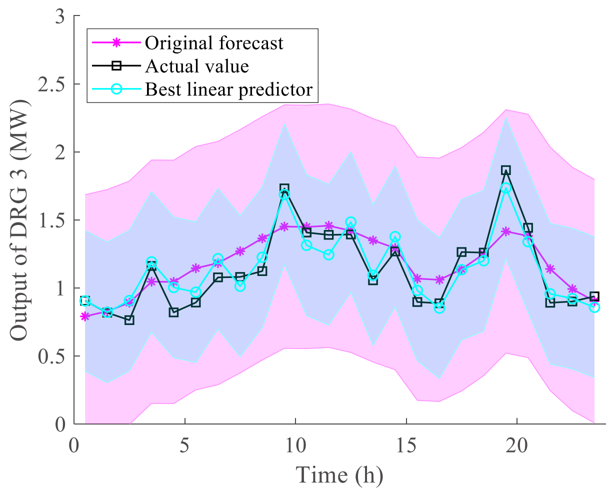

Figure 3. Output of DRG 3.

The uncertainty sets of the output of DRG 3 before and after purchasing distributed predictive information are depicted in Figure 3. For simplicity, the uncertainty budget is not plotted. The pink region is the original uncertainty set without using the purchased information. After buying the distributed predictive information, the new uncertainty set (8) is built, which is the blue region in Figure 3. The blue region is smaller than the pink one, meaning that the distributed information helps improve the uncertainty forecast. The actual values are in the blue region, which shows that the new uncertainty set maintains robust.

4.2. Valuation of distributed predictive information

We investigated the value of distributed predictive information by studying how the optimal solution of problem (9) changes under different information payments $C_i, i \in \mathcal{I}$. The information purchase decisions are shown in Figure 4, and the total cost is depicted in Figure 5.

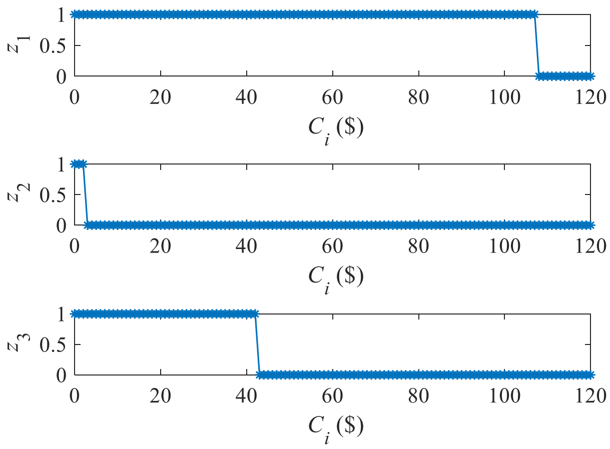

Figure 4. Purchase decisions under different C_i.

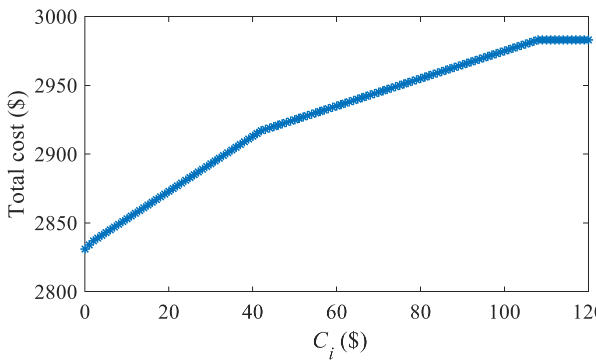

Figure 5. Total cost under different C_i.

As Figure 4 shows, the system operator will buy the distributed predictive information from DRG 1, DRG 2, and DRG 3 if the payment C_i is no larger than US$107, US$2, and US$42, respectively. These payment thresholds can be regarded as the value of the corresponding distributed predictive information. The output of DRG 1 has the largest variance $\sigma_{U,1}^2 = 0.08$ MW^2, and the value of its distributed predictive information (US$107) is also the largest. On the contrary, the output of DRG 2 has the lowest variance and its distributed prediction has the lowest information value. This indicates that the predictive information of a DRG with more fluctuating outputs tends to be more valuable, and the operator is willing to pay more for it.

4.3. Comparison

To illustrate the necessity of transforming DDU into DIU, we compare the results obtained by using the proposed method and by the direct adoption of C&CG. Write $\mathcal{Y}(p,r,u)$ in the compact form as:

$$\mathcal{Y}(p,r,u) = \{y: A'y \geq B'u + D'x + q'\}, \tag{23}$$

where A', B', D', and q' are coefficient matrices or vectors. To apply the C&CG algorithm directly, the primary problem (20) in Algorithm 1 is changed into the following problem:

$$\min_{x=(z,p,r),\zeta,y^k} \sum_i C_i z_i + f(p,r) + \zeta, \tag{24a}$$

$$\text{s.t.} \quad (p,r) \in \mathcal{X}, z_i \in \{0,1\}, \forall i \in \mathcal{I}, \tag{24b}$$

$$\zeta \geq c^T y^k, k = 1, \cdots, K, \tag{24c}$$

$$A'y^k \geq B' \cdot u^k + D'x + q', k = 1, \cdots, K. \tag{24d}$$

where $u^k, k = 1, \cdots, K$ are the worst-case scenarios found previously in the algorithm.

The direct adoption of C&CG converges in 14 iterations and the computational time is about 20 seconds. The resulting total cost is US$2,983, higher than the proposed method (US$2,913), and no distributed predictive information is purchased. This shows that the direct application of C&CG fails to deal with the decision-dependent uncertainty set and cannot find the optimal solution.

4.4. Impacts of the renewable penetration rate

We tested the performance of the proposed algorithm under different renewable penetration rates. To this end, the DRG capacities and the standard variances of outputs are changed proportionally. The renewable penetration rate is defined as the ratio of the total capacity of DRGs to the total capacity of all generators. The dispatch results of the proposed algorithm under different renewable penetration rates are listed in Table 2. The purchase decisions of the three DRGs are represented by binary values, i.e., the optimal value of z. The total cost reduction means how much the total cost can decrease by allowing distributed predictive information purchases. According to the results, as the renewable penetration becomes higher, the operator will purchase more information to alleviate the uncertainty. The total cost decreases because more load demands are supplied by renewable generation, and the outputs of generators decrease. Moreover, the total cost reduction increases as the renewable penetration grows, which shows that the proposed method is effective for the robust economic dispatch of high renewable penetration systems. The computation time is acceptable in all cases.

Table 2. Dispatch results of the proposed algorithm under different renewable penetration rates.

Renewable penetration	20%	25%	30%	40%
Purchase decisions	(0,0,0)	(1,0,1)	(1,1,1)	(1,1,1)
Total cost	US$3304	US$2988	US$2558	US$1740
Total cost reduction	0.0%	1.7%	8.2%	12.3%
Computation time	36 seconds	48 seconds	64 seconds	28 seconds

5. Conclusion

In this paper, a robust economic dispatch model is proposed, where the system operator can decide whether to buy predictive information from distributed forecasters to improve the dispatch performance. The purchase decision will affect the range of the uncertainty set, which introduces DDU. Thus, the proposed model belongs to two-stage robust optimisation with DDU. To effectively solve it, the structure of the uncertainty set is analysed, based on which the DDU is transformed into DIU via decomposition, and the problem with DIU is then solved by the C&CG algorithm.

The case studies verify that: First, by shrinking the uncertainty set, distributed predictive information can help to mitigate the conservativeness and enhance the efficiency. Second, the proposed method is effective and the transformation of DDU into DIU is necessary before applying the C&CG algorithm. Otherwise, the predictive information cannot be utilised. Third, the distributed predictive information of the DRG output with a larger variance tends to be more valuable because it has a larger impact on the uncertainty set range.

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Notes on contributors

Dr Rui Xie received her B.E. degree in Electrical Engineering, B.Sc. degree in Mathematics in 2017, and Ph.D. degree in Electrical Engineering in 2022 from Tsinghua University, Beijing, China. She is currently a postdoctoral fellow with the Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Hong Kong SAR. Her current research interests include stochastic optimisation problems in power systems.

Mr Tao Tan received his B.E. degree in Huazhong University of Science & Technology and University of Birmingham, and master's degree from the University of Michigan. He is currently a Ph.D. candidate with the Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong. His research interest is the optimisation of energy systems.

Prof Yue Chen received her B.E. degree in Electrical Engineering from Tsinghua University, Beijing, China, in 2015, B.S. degree in Economics from Peking University, Beijing, China, in 2017, and Ph.D. degree in Electrical Engineering from Tsinghua University, in 2020. She is currently a Vice-Chancellor Assistant Professor with the Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Hong Kong SAR. Her research interests include optimisation, game theory, and mathematical economics, with applications in smart grid and integrated energy systems. She is currently an Associate Editor for IEEE Transactions on Smart Grid, IEEE Power Engineering Letters, and IET Renewable Power Generation.

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